Electronic Circuits-1

Frequency Response of Amplifier

Prof.Dr.Eng. Ahmad Rateb Al-Najjar

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2. FET Amplifier
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Frequency Response of Amplifier

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Frequency Response of Amplifier

$$\boxed{a = b^x, \qquad x = \log_b a}$$

$$x = \log_{10} 10{,}000 = 4$$

$$a = b^x = (10)^2 = 100$$

$$x = \log_b a = \log_{10} 100 = 2$$

$$\boxed{\text{Common logarithm:} \quad x = \log_{10} a}$$

$$\boxed{\text{Natural logarithm:} \quad y = \log_e a}$$

The two are related by

$$\boxed{\log_e a = 2.3 \log_{10} a}$$

TABLE 9.1

$\log_{10} 10^0$	$= 0$
$\log_{10} 10$	$= 1$
$\log_{10} 100$	$= 2$
$\log_{10} 1{,}000$	$= 3$
$\log_{10} 10{,}000$	$= 4$
$\log_{10} 100{,}000$	$= 5$
$\log_{10} 1{,}000{,}000$	$= 6$
$\log_{10} 10{,}000{,}000$	$= 7$
$\log_{10} 100{,}000{,}000$	$= 8$
etc.	

$$\boxed{\log_{10} 1 = 0}$$

$$\boxed{\log_{10} \frac{a}{b} = \log_{10} a - \log_{10} b}$$

$$\boxed{\log_{10} \frac{1}{b} = -\log_{10} b}$$

$$\boxed{\log_{10} ab = \log_{10} a + \log_{10} b}$$

$$\boxed{G = \log_{10} \frac{P_2}{P_1}} \quad \text{bel}$$

$$\boxed{G_{\text{dB}} = 10 \log_{10} \frac{P_2}{P_1}} \quad \text{dB}$$

Frequency Response of Amplifier

EXAMPLE 9.6 The input power to a device is 10,000 W at a voltage of 1000 V. The output power is 500 W and the output impedance is 20 Ω.

a. Find the power gain in decibels.
b. Find the voltage gain in decibels.
c. Explain why parts (a) and (b) agree or disagree.

Solution:

a. $$G_{dB} = 10\log_{10}\frac{P_o}{P_i} = 10\log_{10}\frac{500\ \text{W}}{10\ \text{kW}} = 10\log_{10}\frac{1}{20} = -10\log_{10}20$$
$$= -10(1.301) = \mathbf{-13.01\ dB}$$

b. $$G_v = 20\log_{10}\frac{V_o}{V_i} = 20\log_{10}\frac{\sqrt{PR}}{1000} = 20\log_{10}\frac{\sqrt{(500\ \text{W})(20\ \Omega)}}{1000\ \text{V}}$$
$$= 20\log_{10}\frac{100}{1000} = 20\log_{10}\frac{1}{10} = -20\log_{10}10 = \mathbf{-20\ dB}$$

c. $$R_i = \frac{V_i^2}{P_i} = \frac{(1\ \text{kV})^2}{10\ \text{kW}} = \frac{10^6}{10^4} = \mathbf{100\ \Omega} \neq R_o = \mathbf{20\ \Omega}$$

Comparing $A_v = \frac{V_o}{V_i}$ *to dB*

Voltage Gain, V_o/V_i	dB Level
0.5	−6
0.707	−3
1	0
2	6
10	20
40	32
100	40
1000	60
10,000	80
etc.	

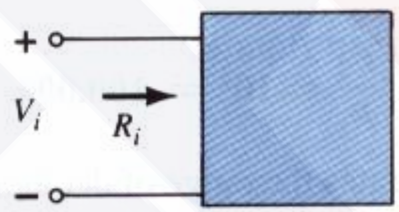
$$|A_{v_T}| = |A_{v_1}| \cdot |A_{v_2}| \cdot |A_{v_3}| \cdots |A_{v_n}|$$

$$\boxed{G_{dBm} = 10\log_{10}\frac{P_2}{1\ \text{mW}}\bigg|_{600\ \Omega}}\quad \text{dBm}$$

Frequency Response of Amplifier

$$G_{dB} = 10\log_{10}\frac{P_2}{P_1} = 10\log_{10}\frac{V_2^2/R_i}{V_1^2/R_i} = 10\log_{10}\left(\frac{V_2}{V_1}\right)^2$$

$$\boxed{G_{dB} = 20\log_{10}\frac{V_2}{V_1}} \qquad \text{dB}$$



Applying the proper logarithmic relationship results in

$$G_v = 20\log_{10}|A_{v_T}| = 20\log_{10}|A_{v_1}| + 20\log_{10}|A_{v_2}| + 20\log_{10}|A_{v_3}| + \cdots + 20\log_{10}|A_{v_n}| \qquad \text{(db)}$$

In words, the equation states that the decibel gain of a cascaded system is simply the sum of the decibel gains of each stage, that is,

$$\boxed{G_{dB_T} = G_{dB_1} + G_{dB_2} + G_{dB_3} + \cdots + G_{dB_n}} \qquad \text{dB}$$

$$P_{o_{mid}} = \frac{|V_o^2|}{R_o} = \frac{|A_{v_{mid}}V_i|^2}{R_o}$$

Frequency Response of Amplifier

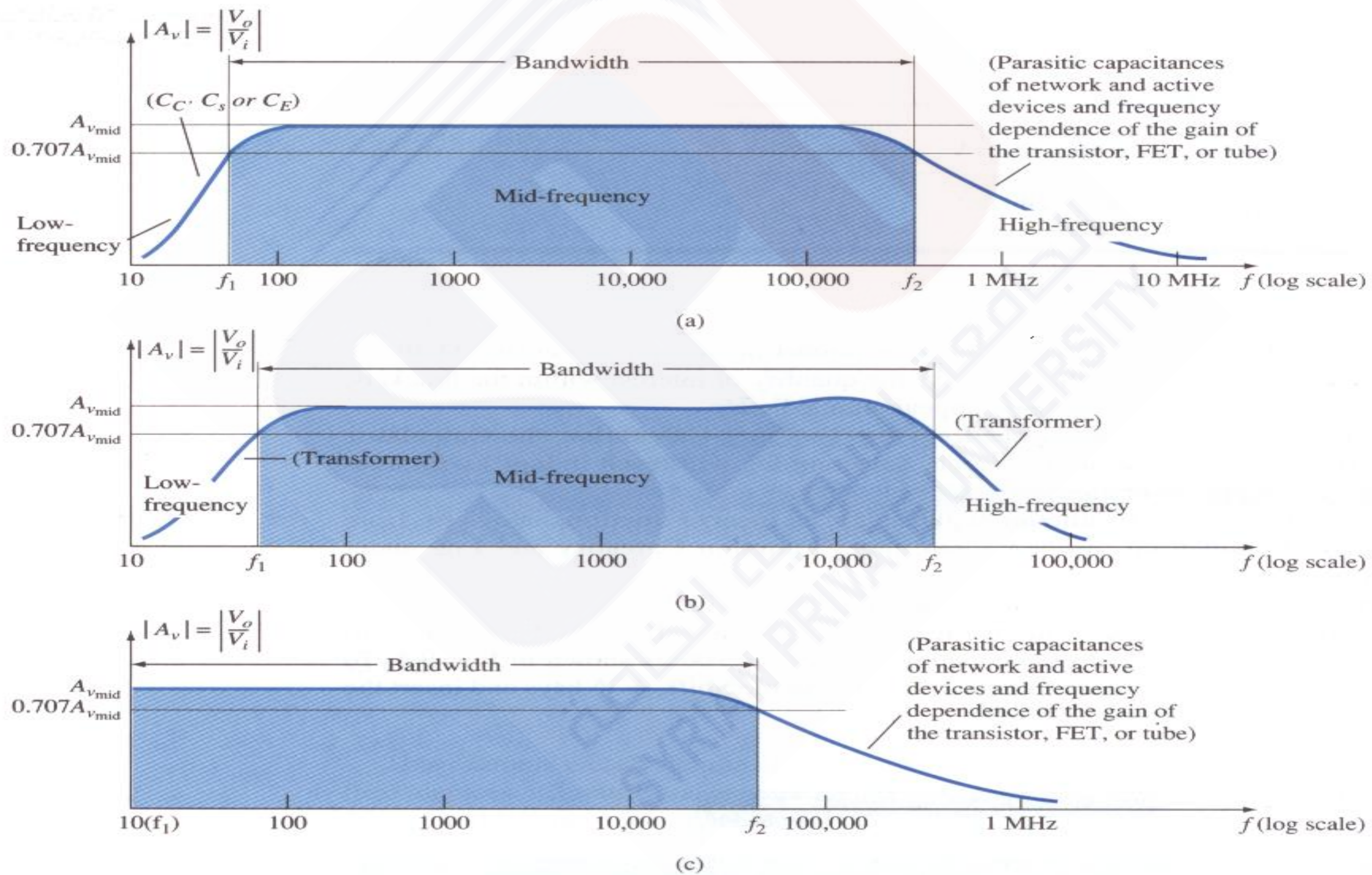
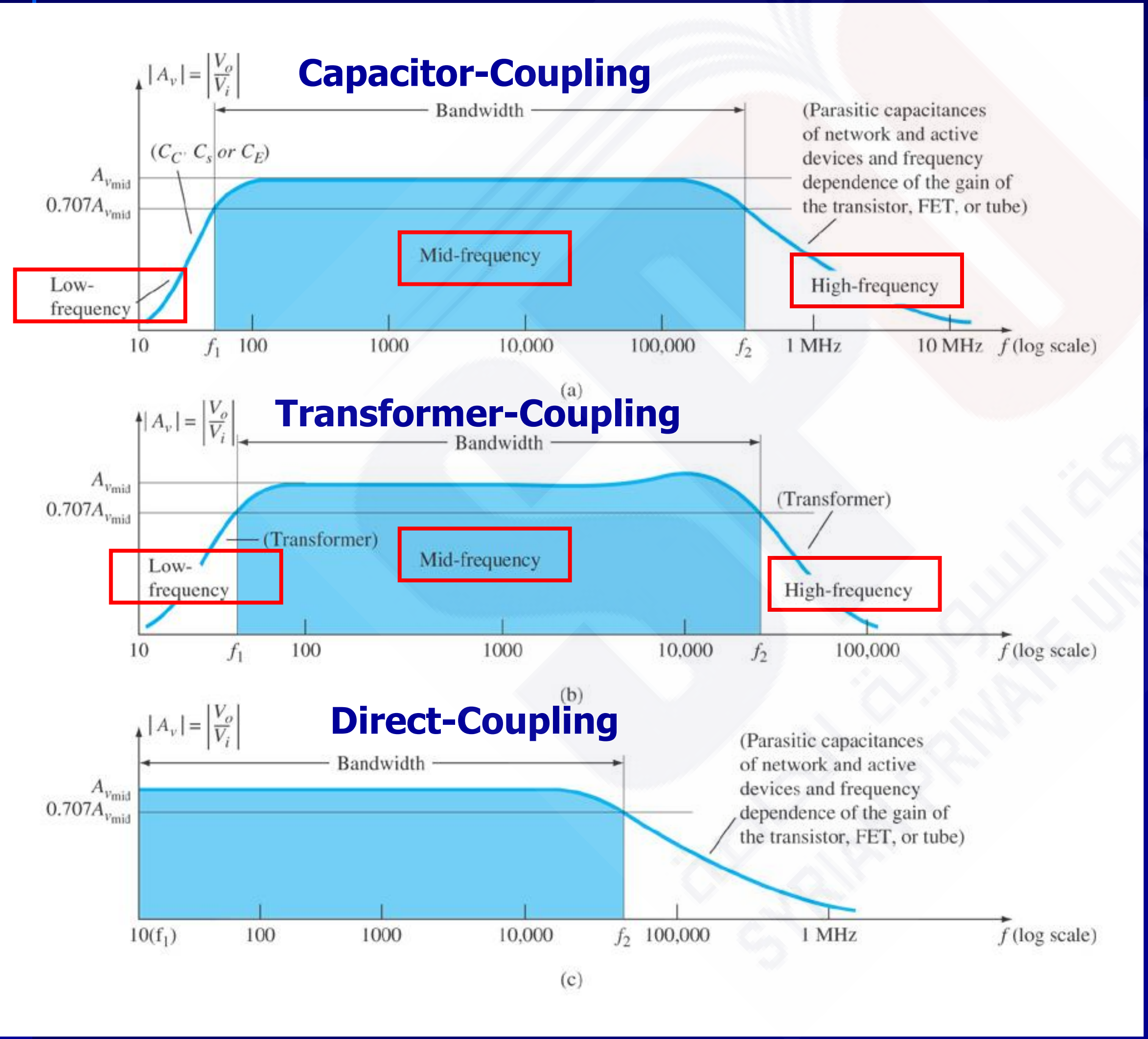


FIG. 9.5

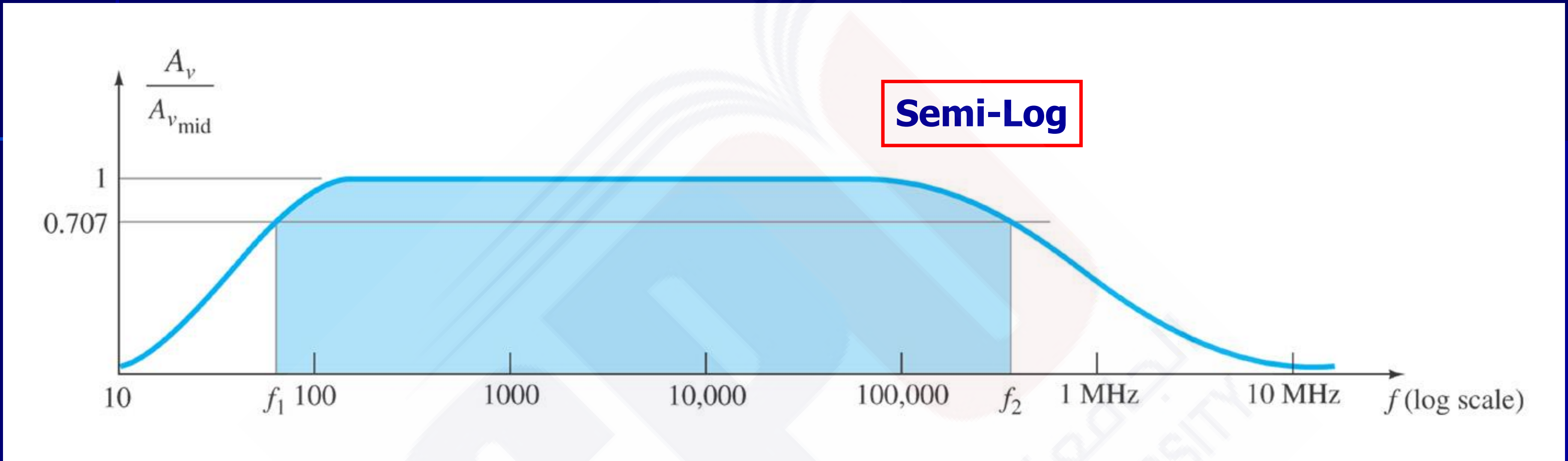
Gain versus frequency: (a) RC-coupled amplifiers; (b) transformer-coupled amplifiers; (c) direct-coupled amplifiers.

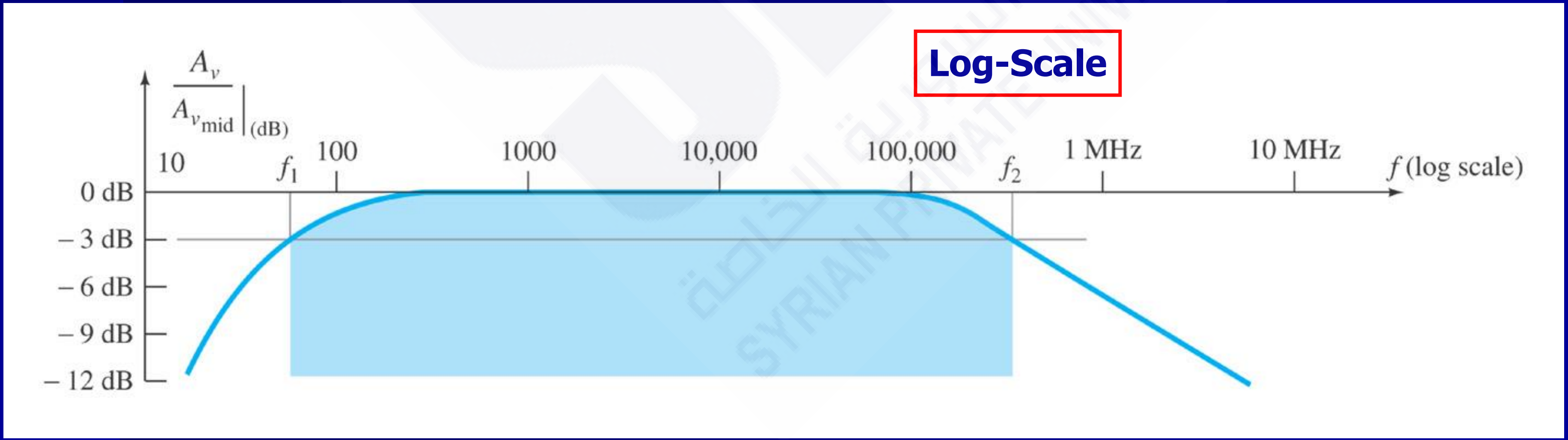
General Frequency Considerations



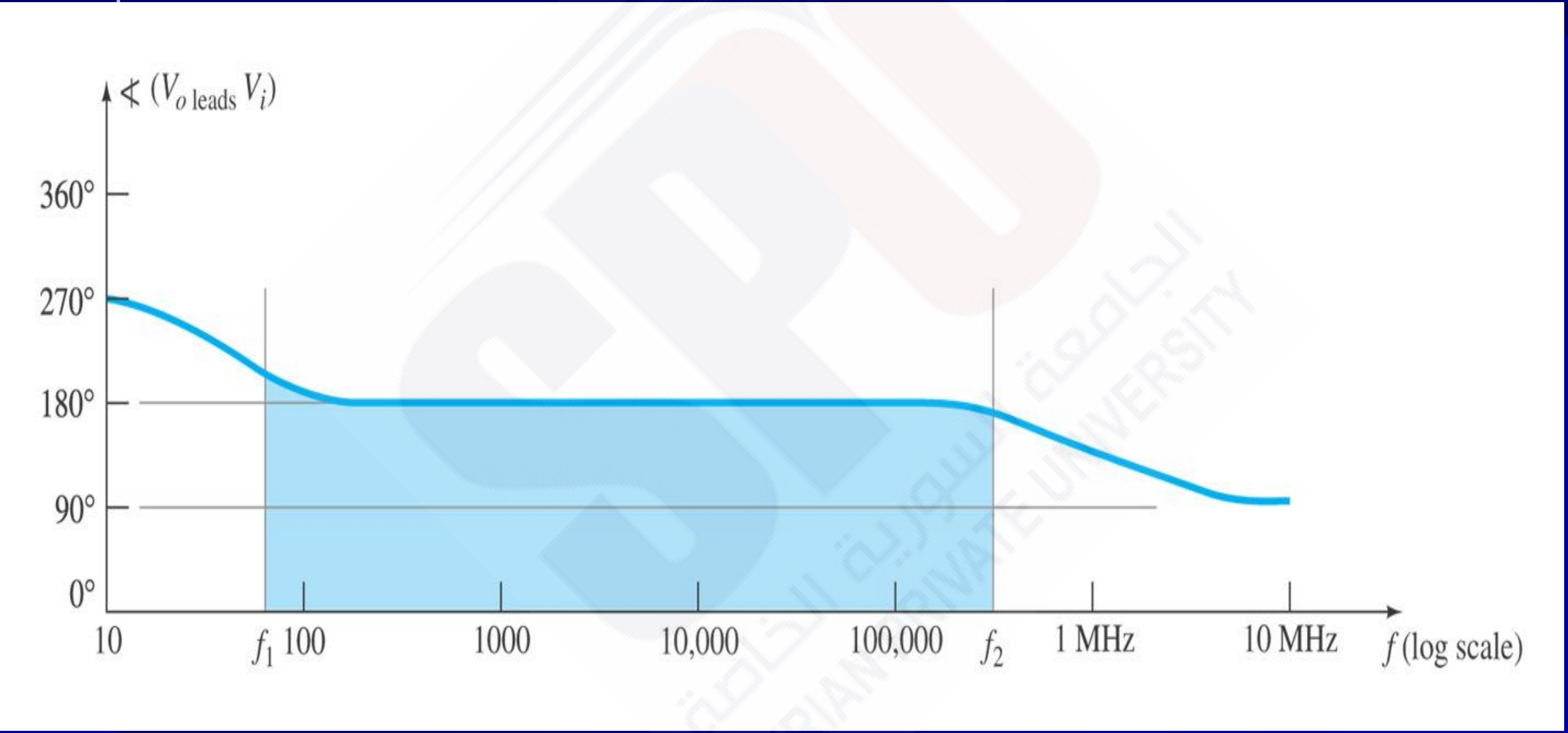
3- types of coupling

General Frequency Considerations

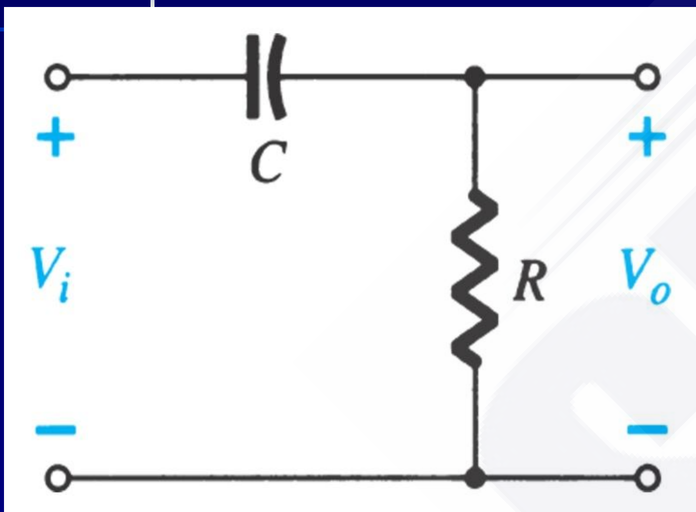


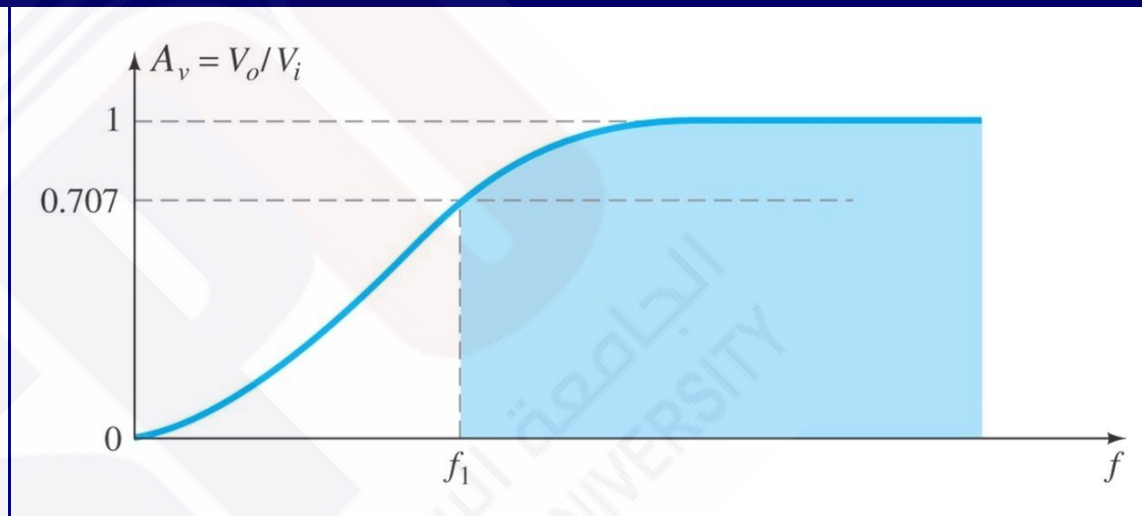


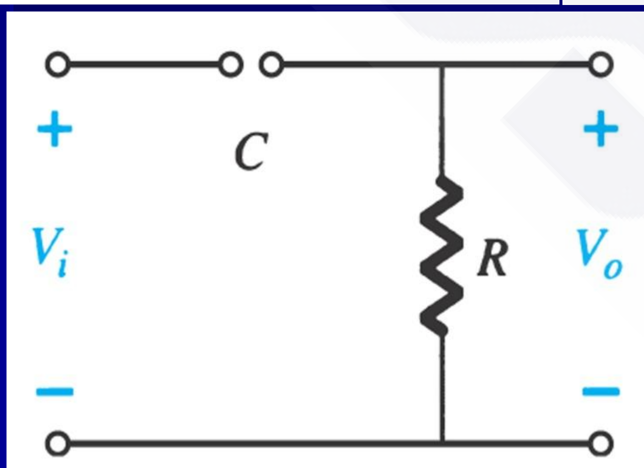
Phase considerations

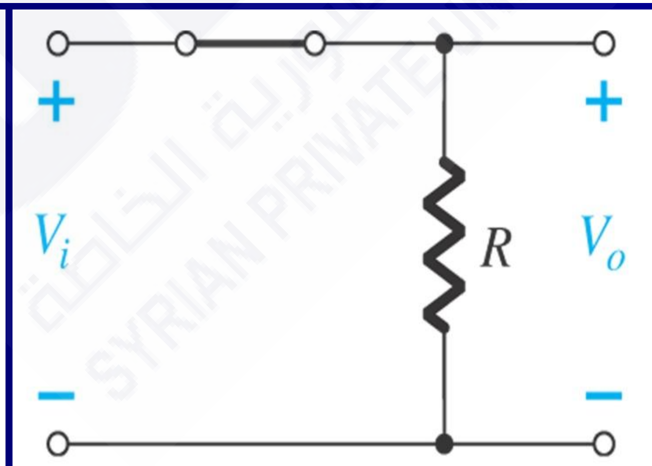


Low-Frequency Analysis & Bode-Plot









$$f_1 = \frac{1}{2\pi CR}$$

manner:

$$\mathbf{V_o} = \frac{\mathbf{RV_i}}{\mathbf{R + X_C}}$$

with the magnitude of V_o determined by

$$V_o = \frac{RV_i}{\sqrt{R^2 + X_C^2}}$$

For the special case where $X_C = R$,

$$V_o = \frac{RV_i}{\sqrt{R^2X_C^2}} = \frac{RV_i}{\sqrt{R^2 + R^2}} = \frac{RV_i}{\sqrt{2R^2}} = \frac{RV_i}{\sqrt{2}R} = \frac{1}{\sqrt{2}}V_i$$

and

$$\boxed{|A_v| = \frac{V_o}{V_i} = \frac{1}{\sqrt{2}} = 0.707|_{X_C=R}}$$

$$X_C = \frac{1}{2\pi f_1 C} = R$$

and

$$\boxed{f_1 = \frac{1}{2\pi RC}}$$

In terms of logs,

$$G_v = 20\log_{10} A_v = 20\log_{10}\frac{1}{\sqrt{2}} = -3\text{ dB}$$

whereas at $A_v = V_o/V_i = 1$ or $V_o = V_i$ (the maximum value),

$$G_v = 20\log_{10} 1 = 20(0) = 0\text{ dB}$$

Frequency Response of Amplifier

$$A_v = \frac{V_o}{V_i} = \frac{R}{R - jX_C} = \frac{1}{1 - j(X_C/R)} = \frac{1}{1 - j(1/\omega CR)} = \frac{1}{1 - j(1/2\pi fCR)}$$

we obtain, using the frequency defined above,

$$\boxed{A_v = \frac{1}{1 - j(f_1/f)}}$$

In the magnitude and phase form,

$$A_v = \frac{V_o}{V_i} = \underbrace{\frac{1}{\sqrt{1 + (f_1/f)^2}}}_{\text{magnitude of } A_v} \underbrace{\angle \tan^{-1}(f_1/f)}_{\text{phase} \measuredangle \text{ by which } V_o \text{ leads } V_i}$$

For the magnitude when $f = f_1$,

$$|A_v| = \frac{1}{\sqrt{1 + (1)^2}} = \frac{1}{\sqrt{2}} = 0.707 \Rightarrow -3\text{ dB}$$

Frequency Response of Amplifier

In the logarithmic form, the gain in dB is

$$\boxed{A_{v(\text{dB})} = 20 \log_{10} \frac{1}{\sqrt{1 + (f_1/f)^2}}}$$

$$A_{v(\text{dB})} = -20 \log_{10}\left[1 + \left(\frac{f_1}{f}\right)^2\right]^{1/2}$$

$$= -\left(\tfrac{1}{2}\right)(20) \log_{10}\left[1 + \left(\frac{f_1}{f}\right)^2\right]$$

$$= -10 \log_{10}\left[1 + \left(\frac{f_1}{f}\right)^2\right]$$

For frequencies where $f \ll f_1$ or $(f_1/f)^2 \gg 1$, the equation above can be approximated by

$$A_{v(\text{dB})} = -10 \log_{10}\left(\frac{f_1}{f}\right)^2$$

and finally,

$$\boxed{A_{v(\text{dB})} = -20 \log_{10} \frac{f_1}{f}} \quad {}_{f \ll f_1}$$

Frequency Response of Amplifier

Ignoring the condition $f \ll f_1$ for a moment, we find that a plot of Eq. (9.24) on a frequency log scale yields a result very useful for future decibel plots.

$$\text{At } f = f_1: \quad \frac{f_1}{f} = 1 \text{ and } -20\log_{10} 1 = 0\,\text{dB}$$

$$\text{At } f = \tfrac{1}{2} f_1: \quad \frac{f_1}{f} = 2 \text{ and } -20\log_{10} 2 \cong -6\,\text{dB}$$

$$\text{At } f = \tfrac{1}{4} f_1: \quad \frac{f_1}{f} = 4 \text{ and } -20\log_{10} 4 \cong -12\,\text{dB}$$

$$\text{At } f = \tfrac{1}{10} f_1: \quad \frac{f_1}{f} = 10 \text{ and } -20\log_{10} 10 = -20\,\text{dB}$$

Frequency Response of Amplifier

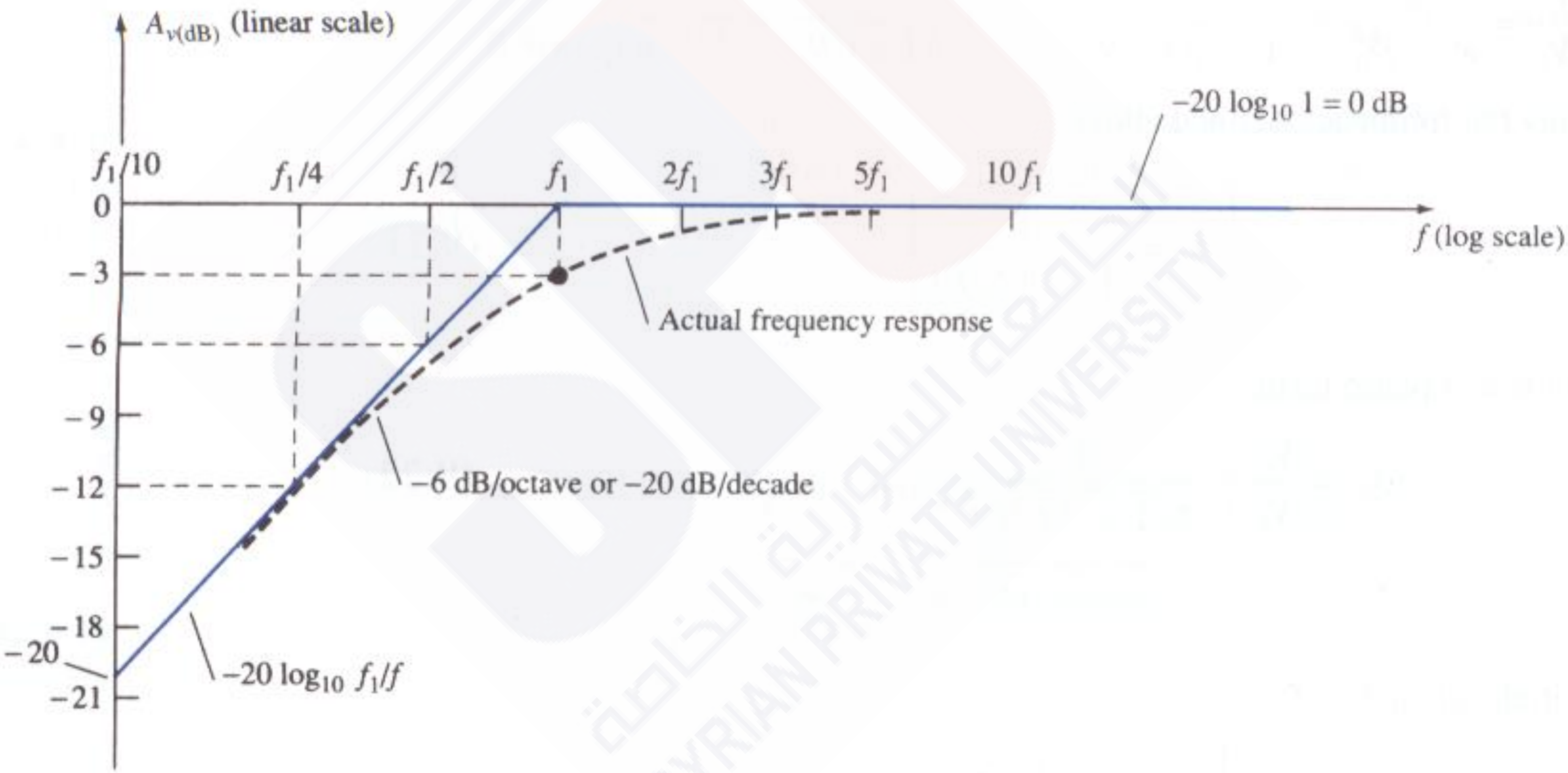
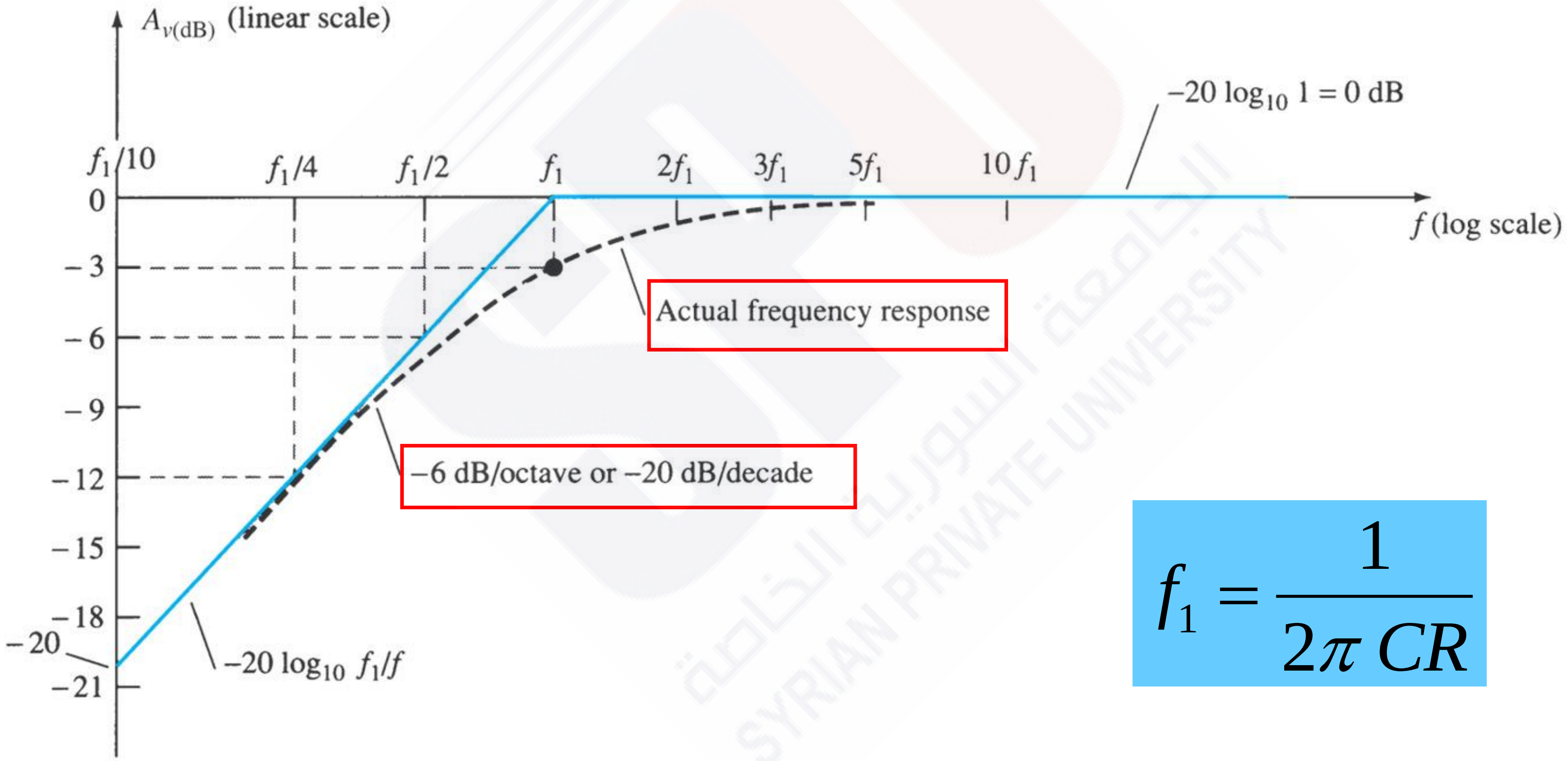


FIG. 9.13

Bode plot for the low-frequency region.

Bode plot for the low-frequency region



$$f_1 = \frac{1}{2\pi\, CR}$$

EXAMPLE 9.8 For the network of Fig. 9.14:

a. Determine the break frequency.
b. Sketch the asymptotes and locate the −3-dB point.
c. Sketch the frequency response curve.

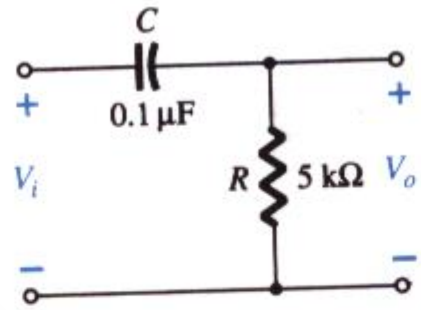


FIG. 9.14
Example 9.8.

Solution:

a. $$f_1 = \frac{1}{2\pi RC} = \frac{1}{(6.28)(5 \times 10^3\ \Omega)(0.1 \times 10^{-6}\ \text{F})}$$

$$\cong \mathbf{318.5\ Hz}$$

b. and c. See Fig. 9.15.

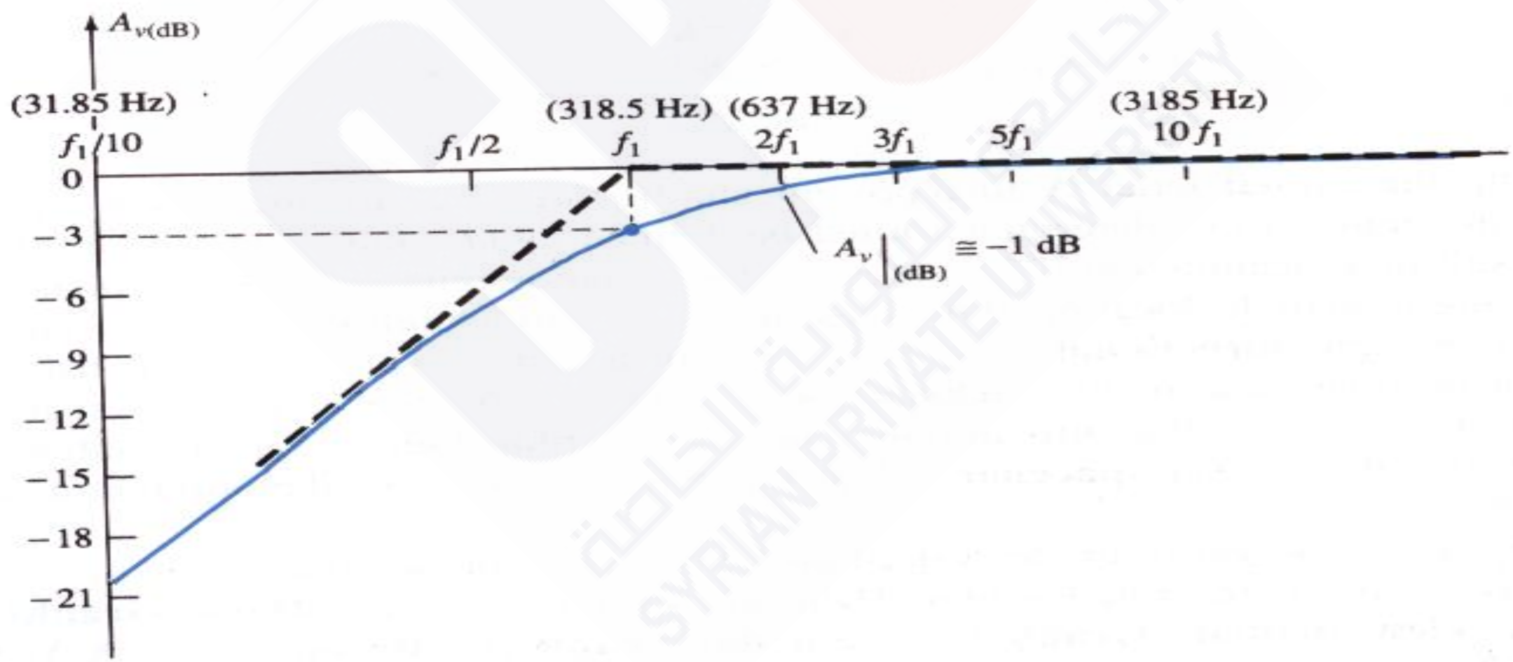


FIG. 9.15
Frequency response for the RC circuit of Fig. 9.14.

Bode plot for the low-frequency region

The gain at any frequency can be determined from the frequency plot in the following manner:

$$A_{v(\text{dB})} = 20 \log_{10} \frac{V_o}{V_i}$$

but

$$\frac{A_{v(\text{dB})}}{20} = \log_{10} \frac{V_o}{V_i}$$

and

$$\boxed{A_v = \frac{V_o}{V_i} = 10^{Av(\text{dB})/20}} \qquad (9.25)$$

For example, if $A_{v(\text{dB})} = -3$ dB,

$$A_v = \frac{V_o}{V_i} = 10^{(-3/20)} = 10^{(-0.15)} \cong 0.707 \qquad \text{as expected}$$

The quantity $10^{-0.15}$ is determined using the 10^x function found on most scientific calculators.

From Fig. 9.15, $A_{v(\text{dB})} \cong -1$ dB at $f = 2f_1 = 637$ Hz. The gain at this point is

$$A_v = \frac{V_o}{V_i} = 10^{A_{v(\text{dB})}/20} = 10^{(-1/20)} = 10^{(-0.05)} = 0.891$$

and

$$V_o = 0.891V_i$$

or V_o is 89.1% of V_i at $f = 637$ Hz.

Frequency Response of Amplifier

The phase angle of θ is determined from

$$\boxed{\theta = \tan^{-1}\frac{f_1}{f}}$$

from Eq. (9.22).

For frequencies $f \ll f_1$,

$$\theta = \tan^{-1}\frac{f_1}{f} \longrightarrow 90^\circ$$

For instance, if $f_1 = 100f$,

$$\theta = \tan^{-1}\frac{f_1}{f} = \tan^{-1}(100) = 89.4^\circ$$

For $f = f_1$,

$$\theta = \tan^{-1}\frac{f_1}{f} = \tan^{-1}1 = 45^\circ$$

For $f \gg f_1$,

$$\theta = \tan^{-1}\frac{f_1}{f} \longrightarrow 0^\circ$$

For instance, if $f = 100f_1$,

$$\theta = \tan^{-1}\frac{f_1}{f} = \tan^{-1}0.01 = 0.573^\circ$$

Frequency Response of Amplifier

A plot of $\theta = \tan^{-1}(f_1/f)$ is provided in Fig. 9.17. If we add the additional 180° phase shift introduced by an amplifier, the phase plot of Fig. 9.8 is obtained. The magnitude and phase response for an *RC* combination have now been established. In Section 9.6, each capacitor of importance in the low-frequency region will be redrawn in an *RC* format and the cutoff frequency for each determined to establish the low-frequency response for the BJT amplifier.

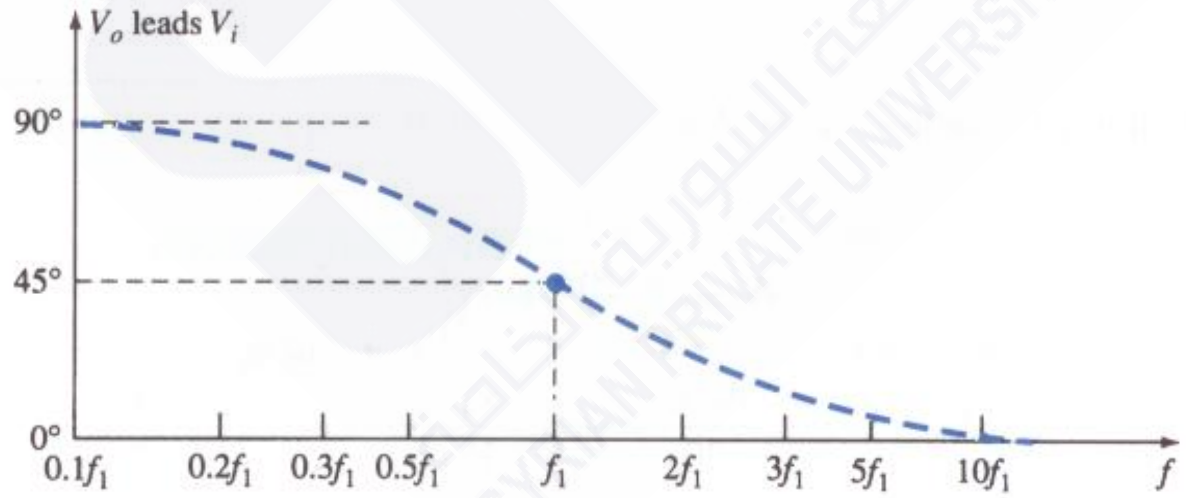


FIG. 9.17

Phase response for the RC circuit of Fig. 9.9.

9.6 LOW-FREQUENCY RESPONSE—BJT AMPLIFIER

The analysis of this section will employ the loaded voltage-divider BJT bias configuration, but the results can be applied to any BJT configuration. It will simply be necessary to find the appropriate equivalent resistance for the *RC* combination. For the network of Fig. 9.18, the capacitors C_s, C_C, and C_E will determine the low-frequency response. We will now examine the impact of each independently in the order listed.

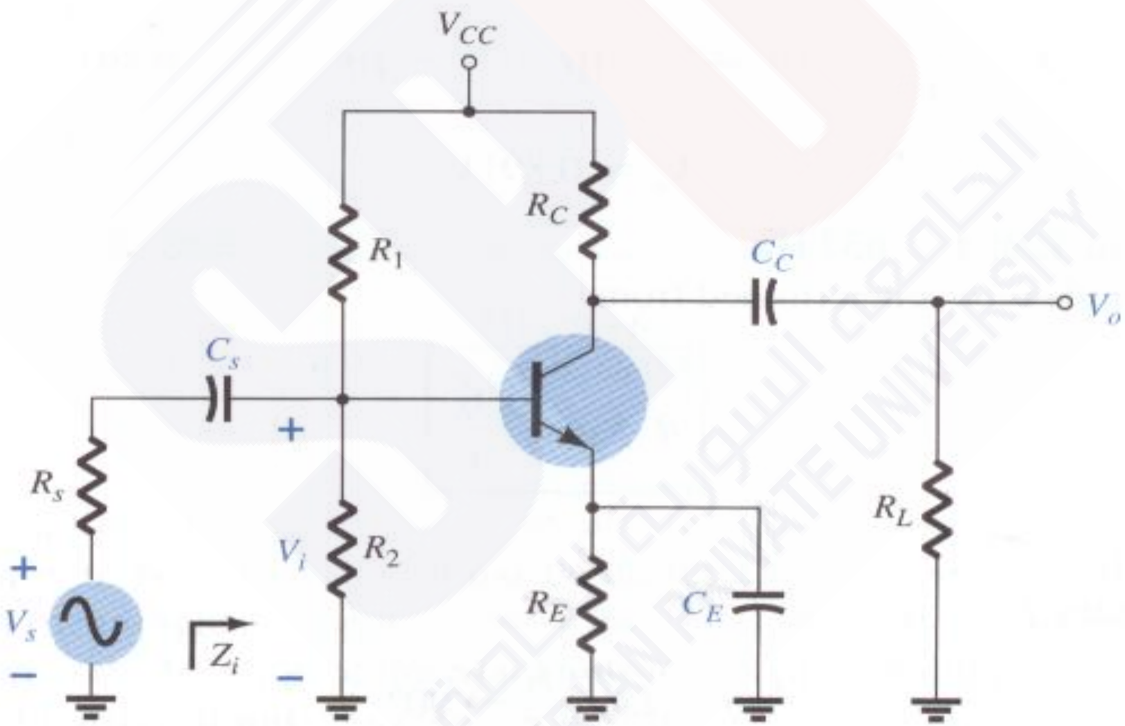


FIG. 9.18
Loaded BJT amplifier with capacitors that affect the low-frequency response.

Low Frequency Analysis-BJT Amp

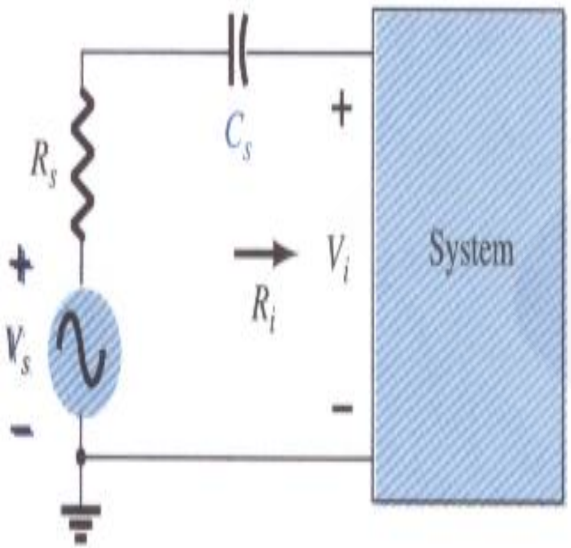


FIG. 9.19
Determining the effect of C_s on the low-frequency response.

C_s Since C_s is normally connected between the applied source and the active device, the general form of the *RC* configuration is established by the network of Fig. 9.19. The total resistance is now and the cutoff frequency as established in Section 9.5 is

$$f_{L_s} = \frac{1}{2\pi(R_s + R_i)C_s} \qquad (9.27)$$

At mid or high frequencies, the reactance of the capacitor will be sufficiently small to permit a short-circuit approximation for the element. The voltage V_i will then be related to V_s by

$$V_i\Big|_{\text{mid}} = \frac{R_i V_s}{R_i + R_s} \qquad (9.28)$$

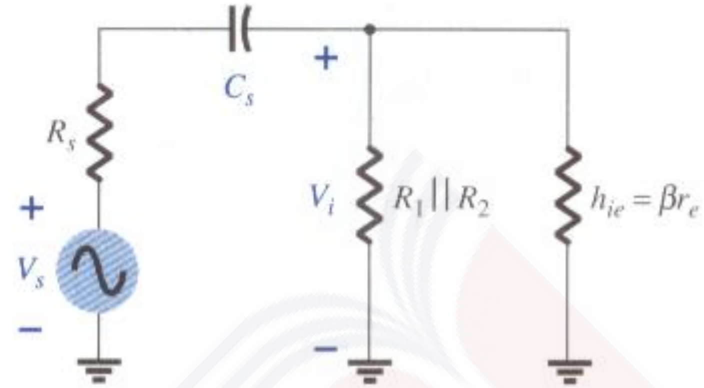


FIG. 9.20
Localized ac equivalent for C_s.

The value of R_i for Eq. (9.27) is determined by

$$\boxed{R_i = R_1 \| R_2 \| \beta r_e} \tag{9.29}$$

The voltage $\mathbf{V}_i$ applied to the input of the active device can be calculated using the voltage-divider rule:

$$\boxed{\mathbf{V}_i = \frac{R_i \mathbf{V}_s}{R_s + R_i - jX_{C_s}}} \tag{9.30}$$

C_C Since the coupling capacitor is normally connected between the output of the active device and the applied load, the *RC* configuration that determines the low-cutoff frequency due to C_C appears in Fig. 9.21. From Fig. 9.21, the total series resistance is now $R_o + R_L$, and the cutoff frequency due to C_C is determined by

$$\boxed{f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C}} \tag{9.31}$$

Low Frequency Analysis-BJT Amp

$$R_o = R_C \| r_o$$

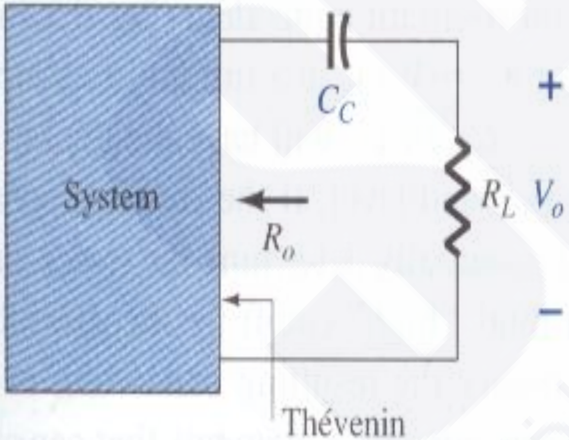


FIG. 9.21

Determining the effect of C_C on the low-frequency response.

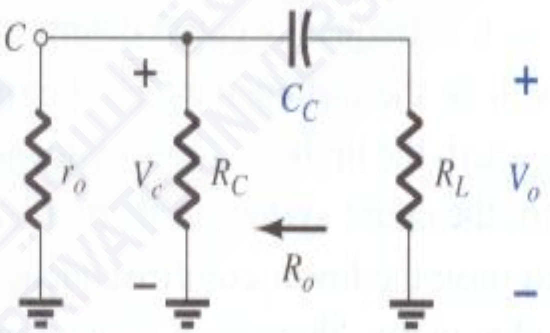


FIG. 9.22

Localized ac equivalent for C_C with $V_i = 0$ V.

Low Frequency Analysis-BJT Amp

$$R_e = R_E \| \left(\frac{R_s'}{\beta} + r_e \right)$$

where $R_s' = R_s \| R_1 \| R_2$.

$$f_{L_E} = \frac{1}{2\pi R_e C_E}$$

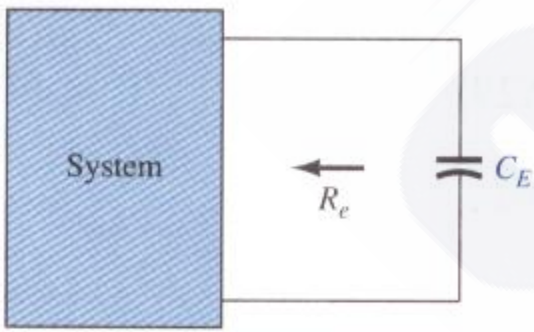


FIG. 9.23
Determining the effect of C_E on the low-frequency response.

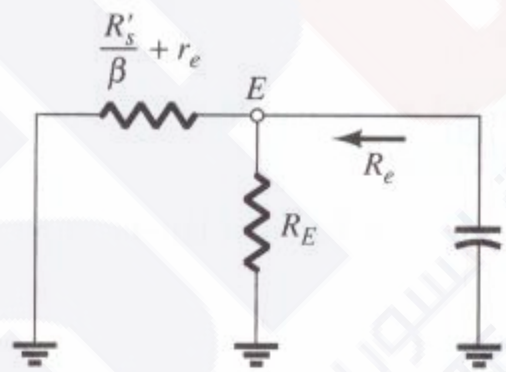


FIG. 9.24
Localized ac equivalent of C_E.

$$A_v = \frac{-R_C}{r_e + R_E}$$

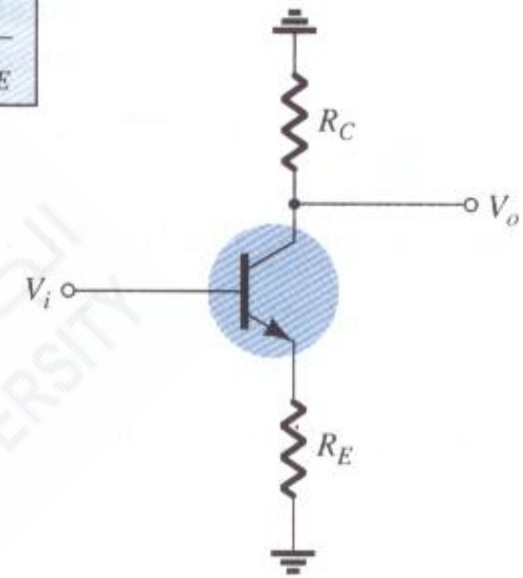


FIG. 9.25
Network employed to describe the effect of C_E on the amplifier gain.

Low Frequency Analysis-BJT Amp

EXAMPLE 9.9

a. Determine the lower cutoff frequency for the network of Fig. 9.18 using the following parameters:

$C_s = 10\,\mu F$, $C_E = 20\,\mu F$, $C_C = 1\,\mu F$
$R_s = 1\,k\Omega$, $R_1 = 40\,k\Omega$, $R_2 = 10\,k\Omega$, $R_E = 2\,k\Omega$, $R_C = 4\,k\Omega$,
$R_L = 2.2\,k\Omega$
$\beta = 100$, $r_o = \infty\,\Omega$, $V_{CC} = 20\,V$

b. Sketch the frequency response using a Bode plot.

Solution:

a. To determine r_e for dc conditions, we use

$$\beta R_E = (100)(2\,k\Omega) = 200\,k\Omega \gg 10R_2 = 100\,k\Omega$$

The result is

$$V_B \cong \frac{R_2 V_{CC}}{R_2 + R_1} = \frac{10\,k\Omega(20\,V)}{10\,k\Omega + 40\,k\Omega} = \frac{200\,V}{50} = 4\,V$$

with

$$I_E = \frac{V_E}{R_E} = \frac{4\,V - 0.7\,V}{2\,k\Omega} = \frac{3.3\,V}{2\,k\Omega} = 1.65\,mA$$

so that

$$r_e = \frac{26\,mV}{1.65\,mA} \cong \mathbf{15.76\,\Omega}$$

and

$$\beta r_e = 100(15.76\,\Omega) = 1576\,\Omega = \mathbf{1.576\,k\Omega}$$

Midband Gain $A_v = \dfrac{V_o}{V_i} = \dfrac{-R_C \| R_L}{r_e} = -\dfrac{(4\,\text{k}\Omega)\|(2.2\,\text{k}\Omega)}{15.76\,\Omega} \cong -90$

The input impedance is given by

$$\begin{aligned} Z_i = R_i &= R_1 \| R_2 \| \beta r_e \\ &= 40\,\text{k}\Omega \| 10\,\text{k}\Omega \| 1.576\,\text{k}\Omega \\ &\cong 1.32\,\text{k}\Omega \end{aligned}$$

and from Fig. 9.26,

$$V_i = \frac{R_i V_s}{R_i + R_s}$$

or

$$\frac{V_i}{V_s} = \frac{R_i}{R_i + R_s} = \frac{1.32\,\text{k}\Omega}{1.32\,\text{k}\Omega + 1\,\text{k}\Omega} = 0.569$$

so that

$$\begin{aligned} A_{v_s} = \frac{V_o}{V_s} &= \frac{V_o}{V_i} \cdot \frac{V_i}{V_s} = (-90)(0.569) \\ &= \mathbf{-51.21} \end{aligned}$$

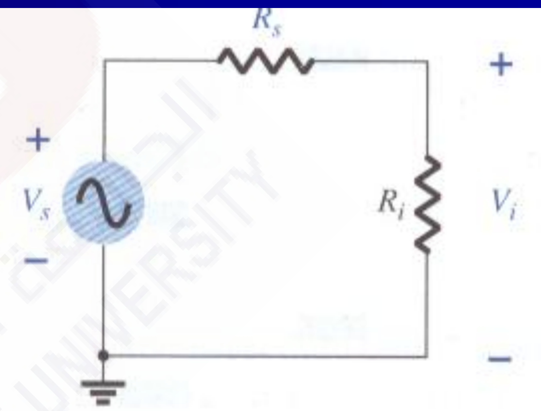


FIG. 9.26
Determining the effect of R_s on the gain A_{v_s}.

$\mathbf{C_s}$ $R_i = R_1 \| R_2 \| \beta r_e = 40\,\text{k}\Omega \| 10\,\text{k}\Omega \| 1.576\,\text{k}\Omega \cong 1.32\,\text{k}\Omega$

$$f_{L_s} = \frac{1}{2\pi(R_s + R_i)C_s} = \frac{1}{(6.28)(1\,\text{k}\Omega + 1.32\,\text{k}\Omega)(10\,\mu\text{F})}$$

$$f_{L_s} \cong \mathbf{6.86\ Hz}$$

9.7 LOW-FREQUENCY RESPONSE—FET AMPLIFIER

The analysis of the FET amplifier in the low-frequency region will be quite similar to that of the BJT amplifier of Section 9.6. There are again three capacitors of primary concern as appearing in the network of Fig. 9.34: C_G, C_C, and C_S. Although Fig. 9.34 will be used to establish the fundamental equations, the procedure and conclusions can be applied to most FET configurations.

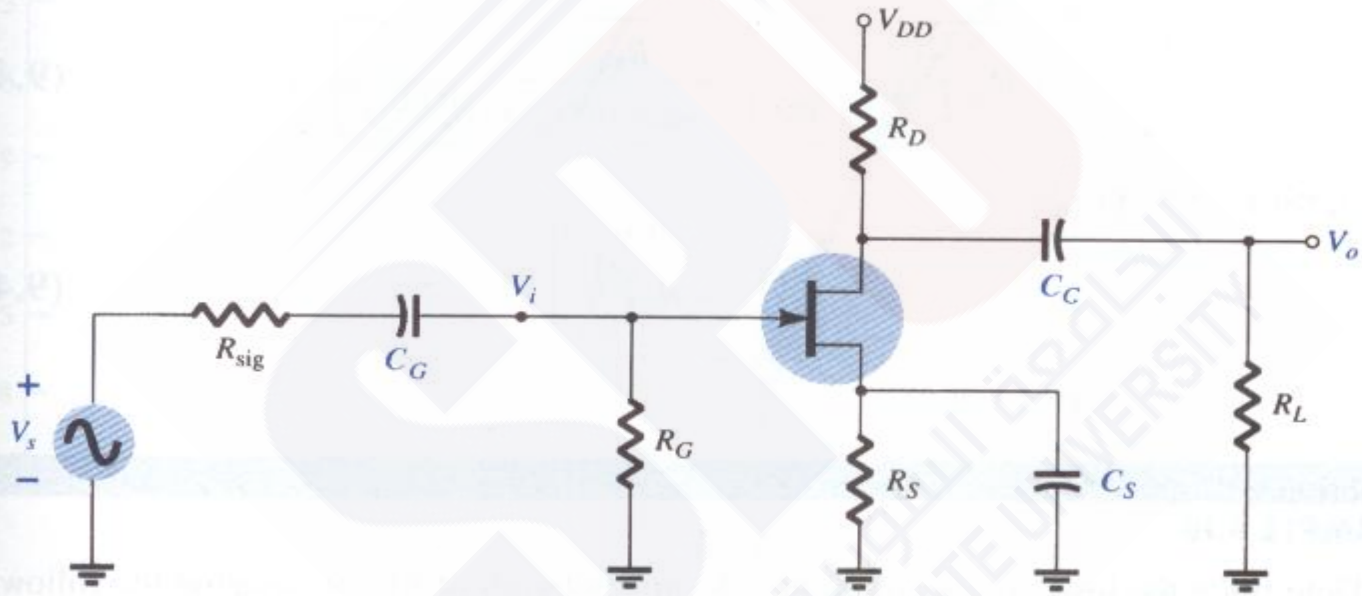


FIG. 9.34
Capacitive elements that affect the low-frequency response of a JFET amplifier.

C_G For the coupling capacitor between the source and the active device, the ac equivalent network is as shown in Fig. 9.35. The cutoff frequency determined by C_G is

$$\boxed{f_{L_G} = \frac{1}{2\pi(R_{sig} + R_i)C_G}} \tag{9.35}$$

Low Frequency Analysis-FET Amp

$$f_{L_G} = \frac{1}{2\pi(R_{sig} + R_i)C_G}$$

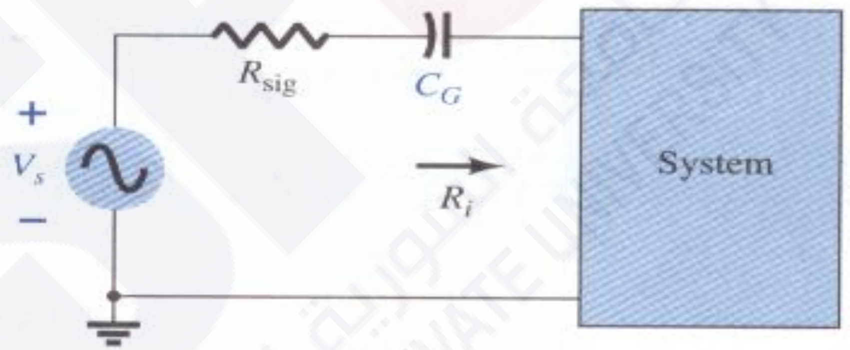


FIG. 9.35

Determining the effect of C_G on the low-frequency response.

which is an exact match of Eq. (9.27). For the network of Fig. 9.34,

$$R_i = R_G$$

Low Frequency Analysis-FET Amp

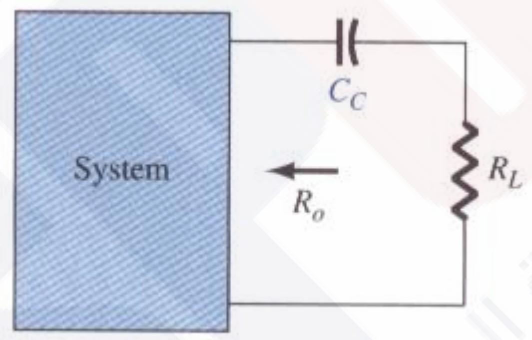


FIG. 9.36
Determining the effect of C_C on the low-frequency response.

$$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C}$$

9.7 LOW-FREQUENCY RESPONSE—FET AMPLIFIER

The analysis of the FET amplifier in the low-frequency region will be quite similar to that of the BJT amplifier of Section 9.6. There are again three capacitors of primary concern as appearing in the network of Fig. 9.34: C_G, C_C, and C_S. Although Fig. 9.34 will be used to establish the fundamental equations, the procedure and conclusions can be applied to most FET configurations.

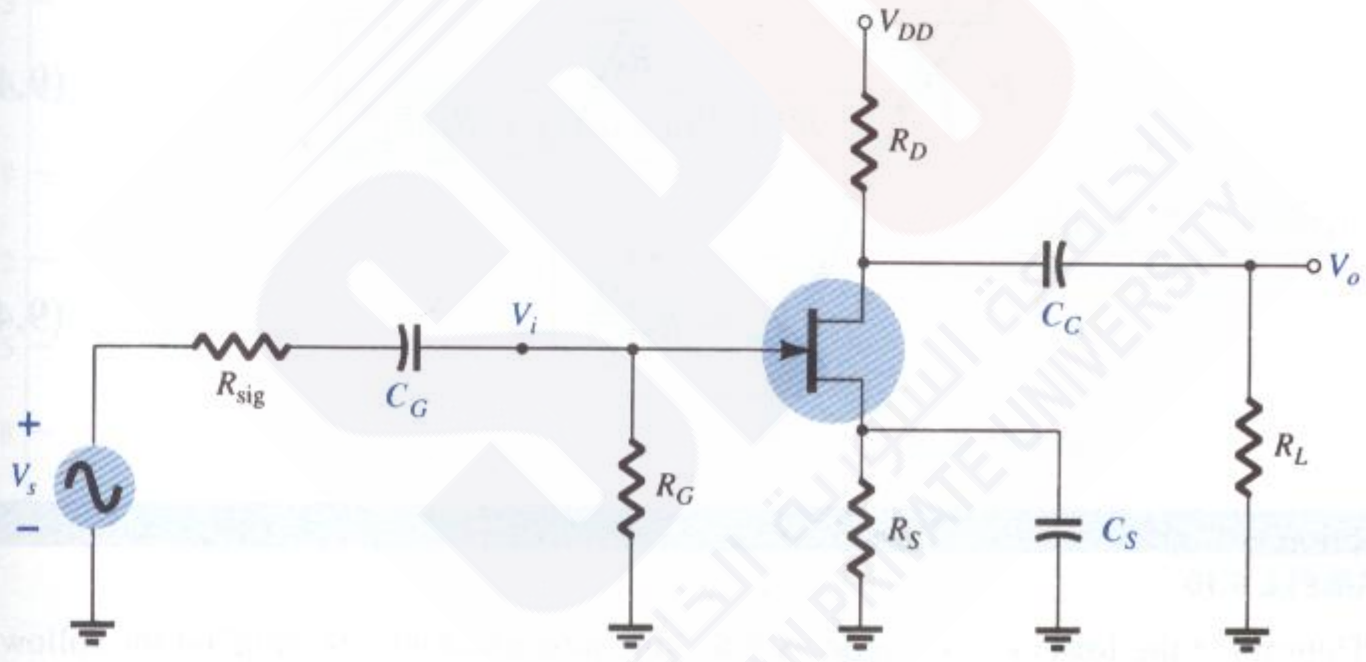


FIG. 9.34

Capacitive elements that affect the low-frequency response of a JFET amplifier.

Low Frequency Analysis-FET Amp

C_G For the coupling capacitor between the source and the active device, the ac equivalent network is as shown in Fig. 9.35. The cutoff frequency determined by C_G is

$$\boxed{f_{L_G} = \frac{1}{2\pi(R_{sig} + R_i)C_G}} \tag{9.35}$$

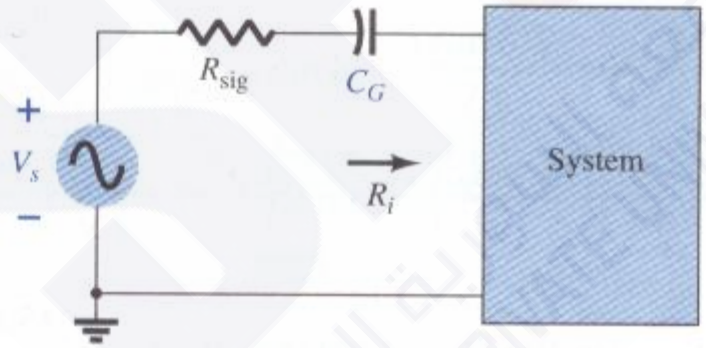


FIG. 9.35
Determining the effect of C_G on the low-frequency response.

which is an exact match of Eq. (9.27). For the network of Fig. 9.34,

$$\boxed{R_i = R_G} \tag{9.36}$$

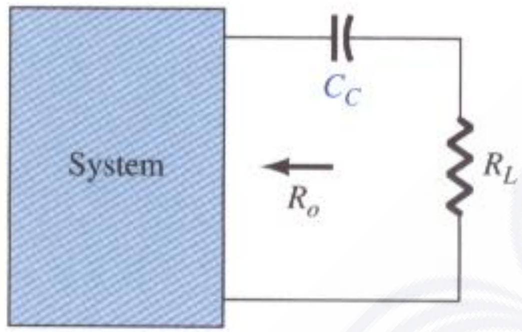


FIG. 9.36
Determining the effect of C_C on the low-frequency response.

$$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C}$$

For the network of Fig. 9.34,

$$R_o = R_D \| r_d \tag{9.38}$$

C_S For the source capacitor C_S, the resistance level of importance is defined by Fig. 9.37. The cutoff frequency is defined by

$$f_{L_S} = \frac{1}{2\pi R_{eq} C_S} \tag{9.39}$$

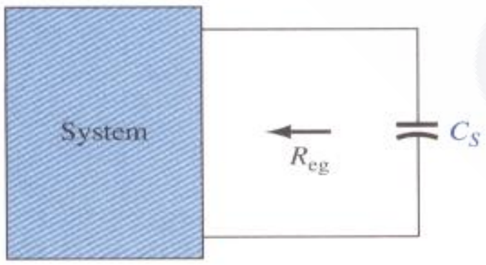


FIG. 9.37
Determining the effect of C_S on the low-frequency response.

For Fig. 9.34, the resulting value of R_{eq} is

$$R_{eq} = \frac{R_S}{1 + R_S(1 + g_m r_d)/(r_d + R_D \| R_L)} \tag{9.40}$$

which for $r_d \cong \infty\,\Omega$ becomes

$$R_{eq} = R_S \| \frac{1}{g_m} \tag{9.41}$$

Low Frequency Analysis-FET Amp

EXAMPLE 9.10

a. Determine the lower cutoff frequency for the network of Fig. 9.34 using the following parameters:

$$C_G = 0.01\,\mu\text{F}, \qquad C_C = 0.5\,\mu\text{F}, \qquad C_S = 2\,\mu\text{F}$$

$$R_{\text{sig}} = 10\,\text{k}\Omega, \quad R_G = 1\,\text{M}\Omega, \quad R_D = 4.7\,\text{k}\Omega, \quad R_S = 1\,\text{k}\Omega, \quad R_L = 2.2\,\text{k}\Omega$$

$$I_{DSS} = 8\,\text{mA}, \qquad V_P = -4\,\text{V} \qquad r_d = \infty\,\Omega, \qquad V_{DD} = 20\,\text{V}$$

b. Sketch the frequency response using a Bode plot.

Solution:

a. DC analysis: Plotting the transfer curve of $I_D = I_{DSS}(1 - V_{GS}/V_P)^2$ and superimposing the curve defined by $V_{GS} = -I_D R_S$ results in an intersection at $V_{GS_Q} = -2\,\text{V}$ and $I_{D_Q} = 2\,\text{mA}$. In addition,

$$g_{m0} = \frac{2I_{DSS}}{|V_P|} = \frac{2(8\,\text{mA})}{4\,\text{V}} = 4\,\text{mS}$$

$$g_m = g_{m0}\left(1 - \frac{V_{GS_Q}}{V_P}\right) = 4\,\text{mS}\left(1 - \frac{-2\,\text{V}}{-4\,\text{V}}\right) = 2\,\text{mS}$$

Low Frequency Analysis-FET Amp

C_G Eq. (9.35): $$f_{L_G} = \frac{1}{2\pi(10\,\text{k}\Omega + 1\,\text{M}\Omega)(0.01\,\mu\text{F})} \cong \mathbf{15.8\,Hz}$$

C_C Eq. (9.37): $$f_{L_C} = \frac{1}{2\pi(4.7\,\text{k}\Omega + 2.2\,\text{k}\Omega)(0.5\,\mu\text{F})} \cong \mathbf{46.13\,Hz}$$

C_S $$R_{eq} = R_S \| \frac{1}{g_m} = 1\,\text{k}\Omega \| \frac{1}{2\,\text{mS}} = 1\,\text{k}\Omega \| 0.5\,\text{k}\Omega = 333.33\,\Omega$$

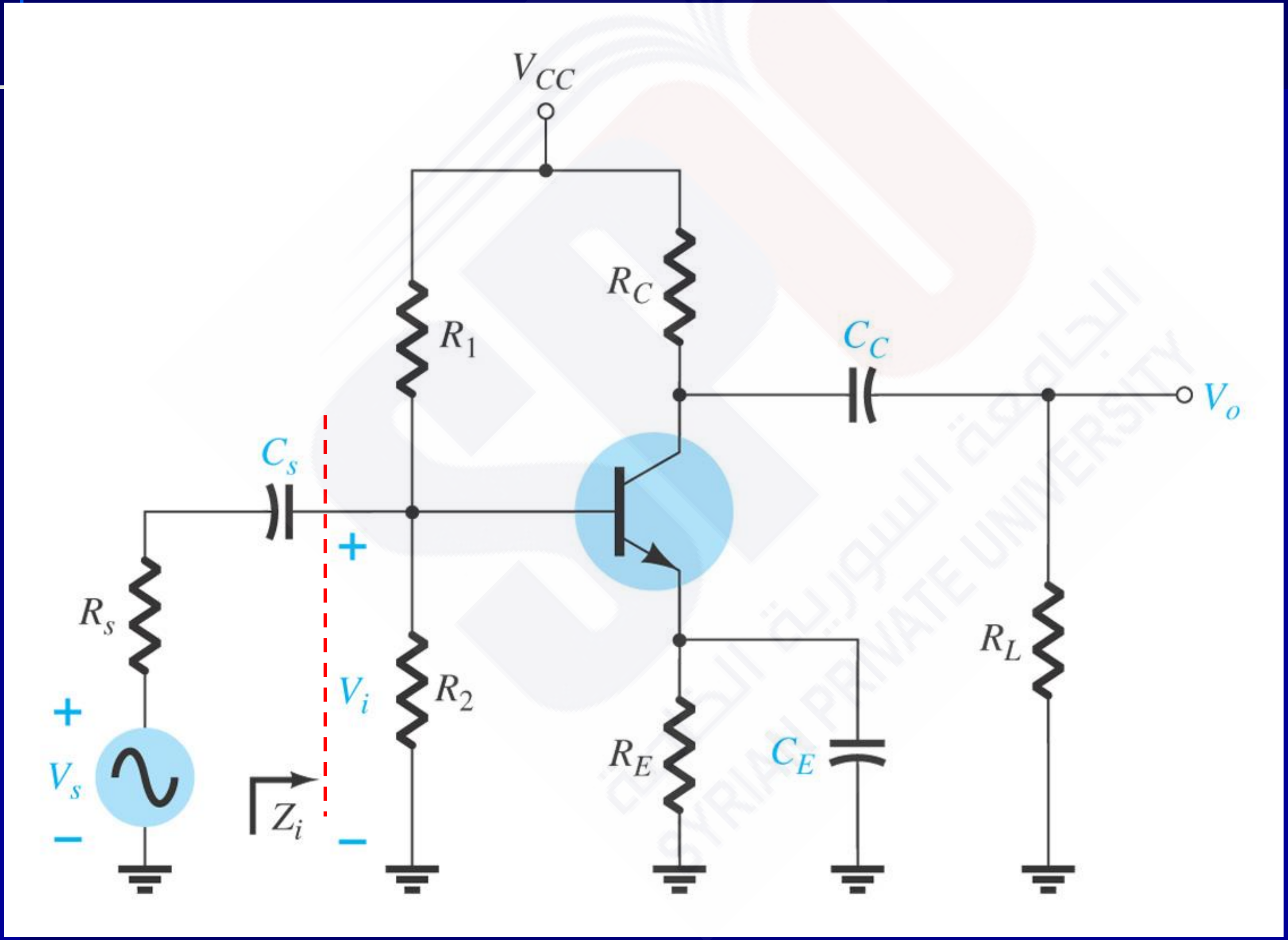
Eq. (9.39): $$f_{L_S} = \frac{1}{2\pi(333.33\,\Omega)(2\,\mu\text{F})} = \mathbf{238.73\,Hz}$$

Since f_{L_s} is the largest of the three cutoff frequencies, it defines the low-cutoff frequency for the network of Fig. 9.34.

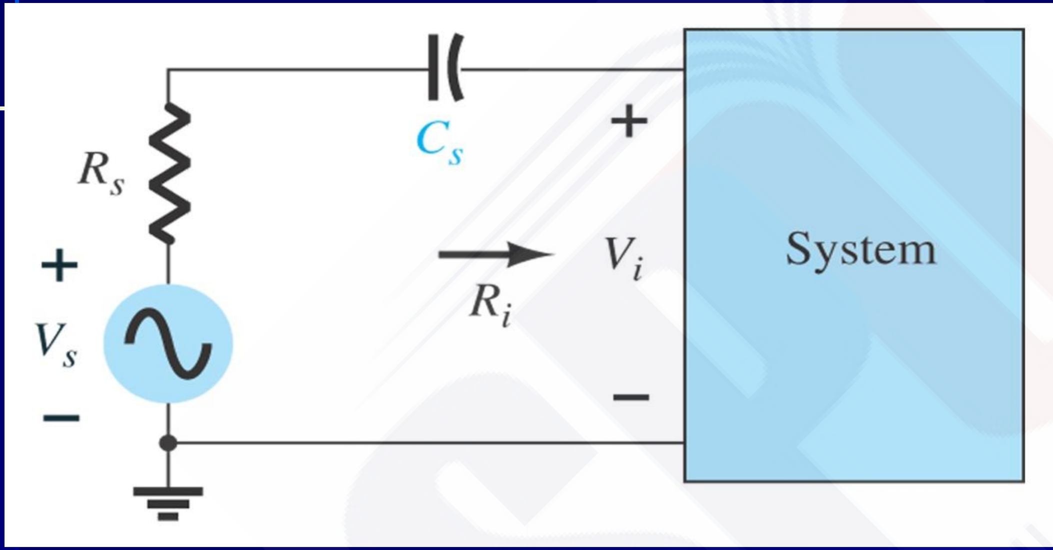
b. The midband gain of the system is determined by

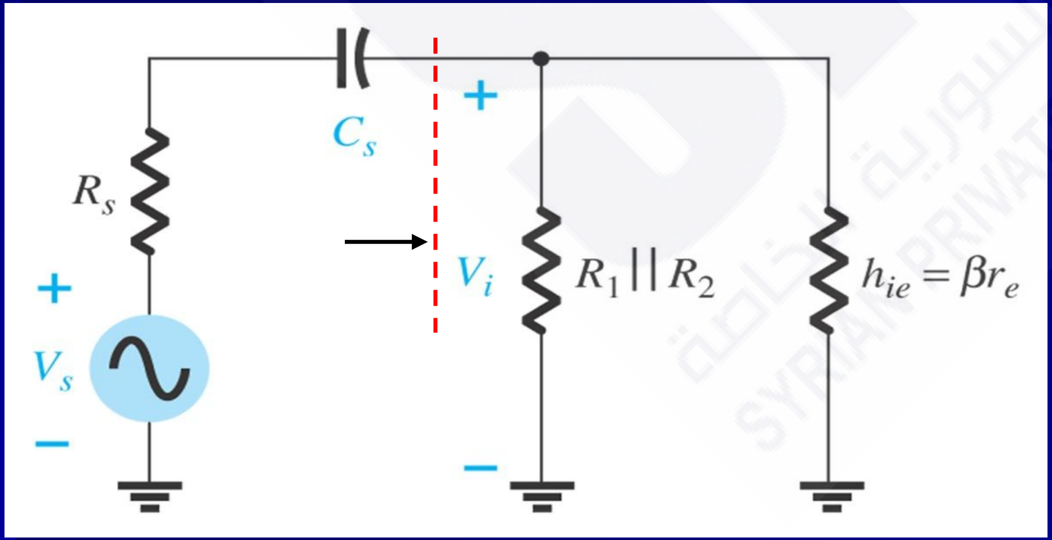
$$\begin{aligned} A_{v_{\text{mid}}} = \frac{V_o}{V_i} &= -g_m(R_D \| R_L) = -(2\,\text{mS})(4.7\,\text{k}\Omega \| 2.2\,\text{k}\Omega) \\ &= -(2\,\text{mS})(1.499\,\text{k}\Omega) \\ &\cong \mathbf{-3} \end{aligned}$$

3.3 Low Frequency Analysis-BJT Amp



Cs Effect

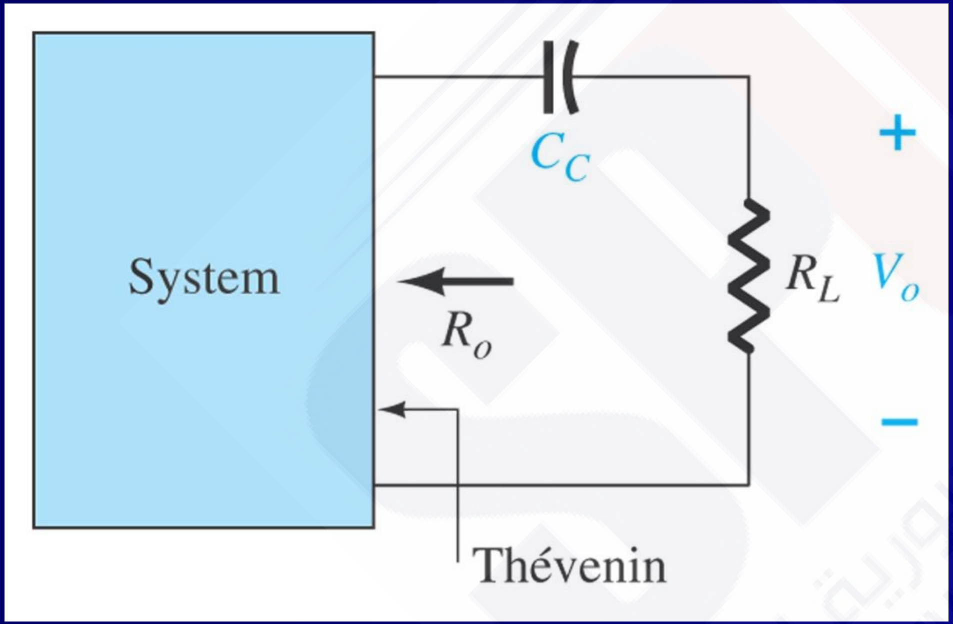




V_i/V_s=?

$$f_C = \frac{1}{2\pi C_s (R_s + R_i)}$$

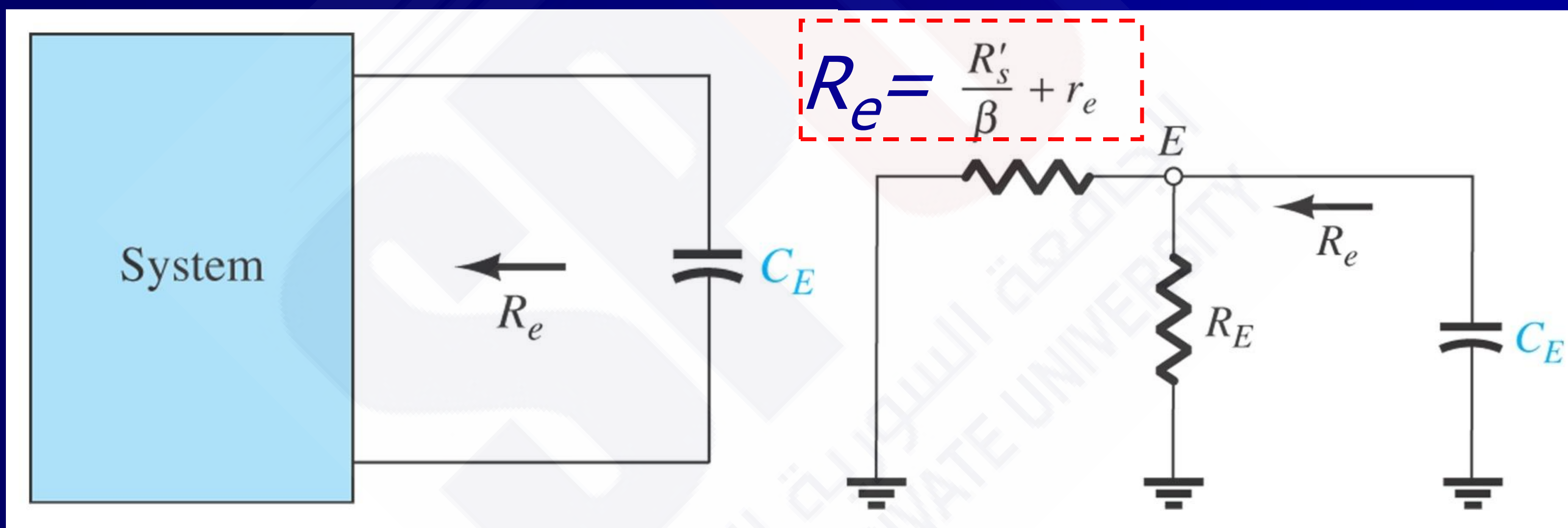
C_C Effect



V_o/V_s=?

$$f_C = \frac{1}{2\pi C_C (R_L + R_o)}$$

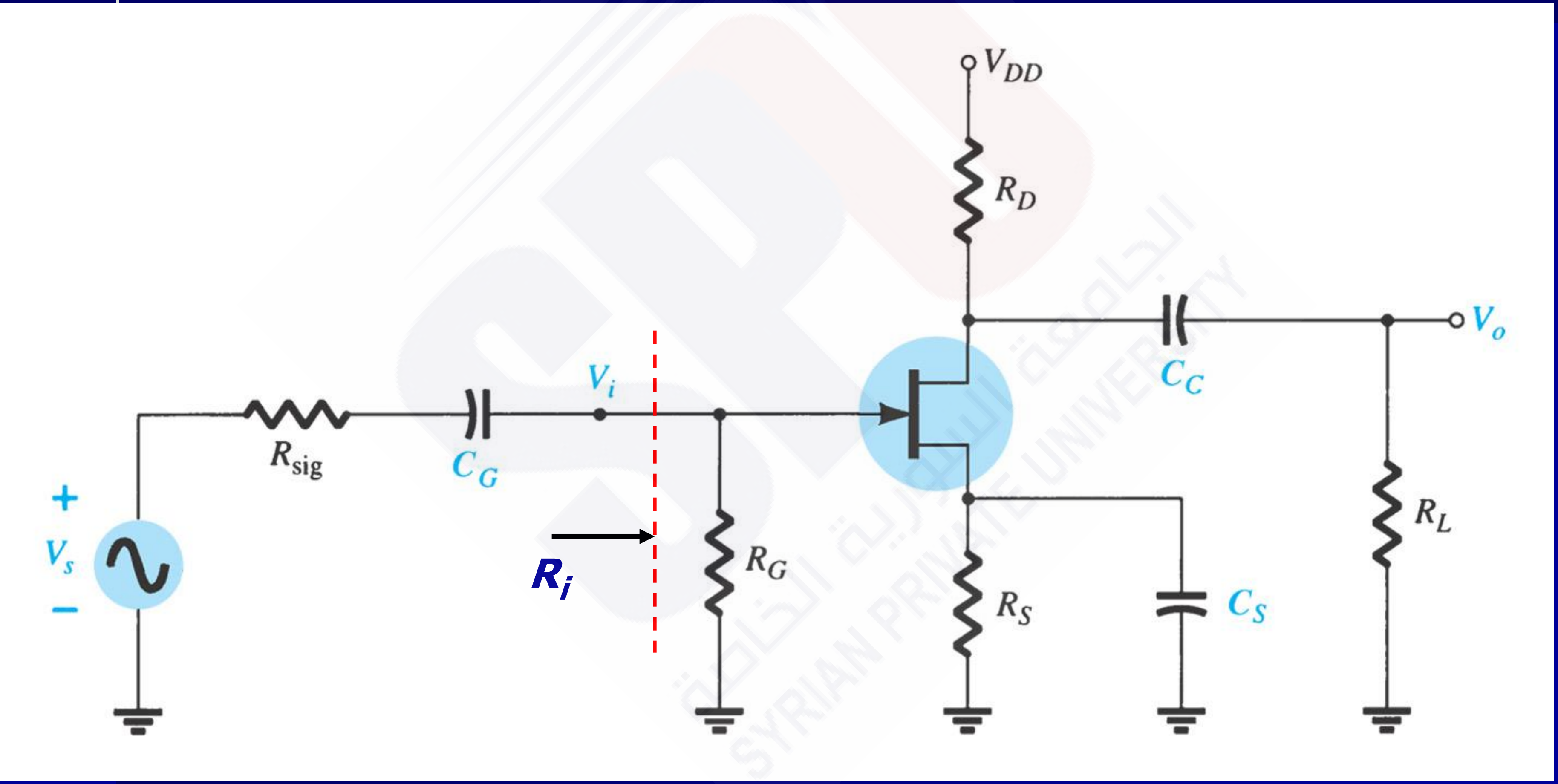
C_E Effect



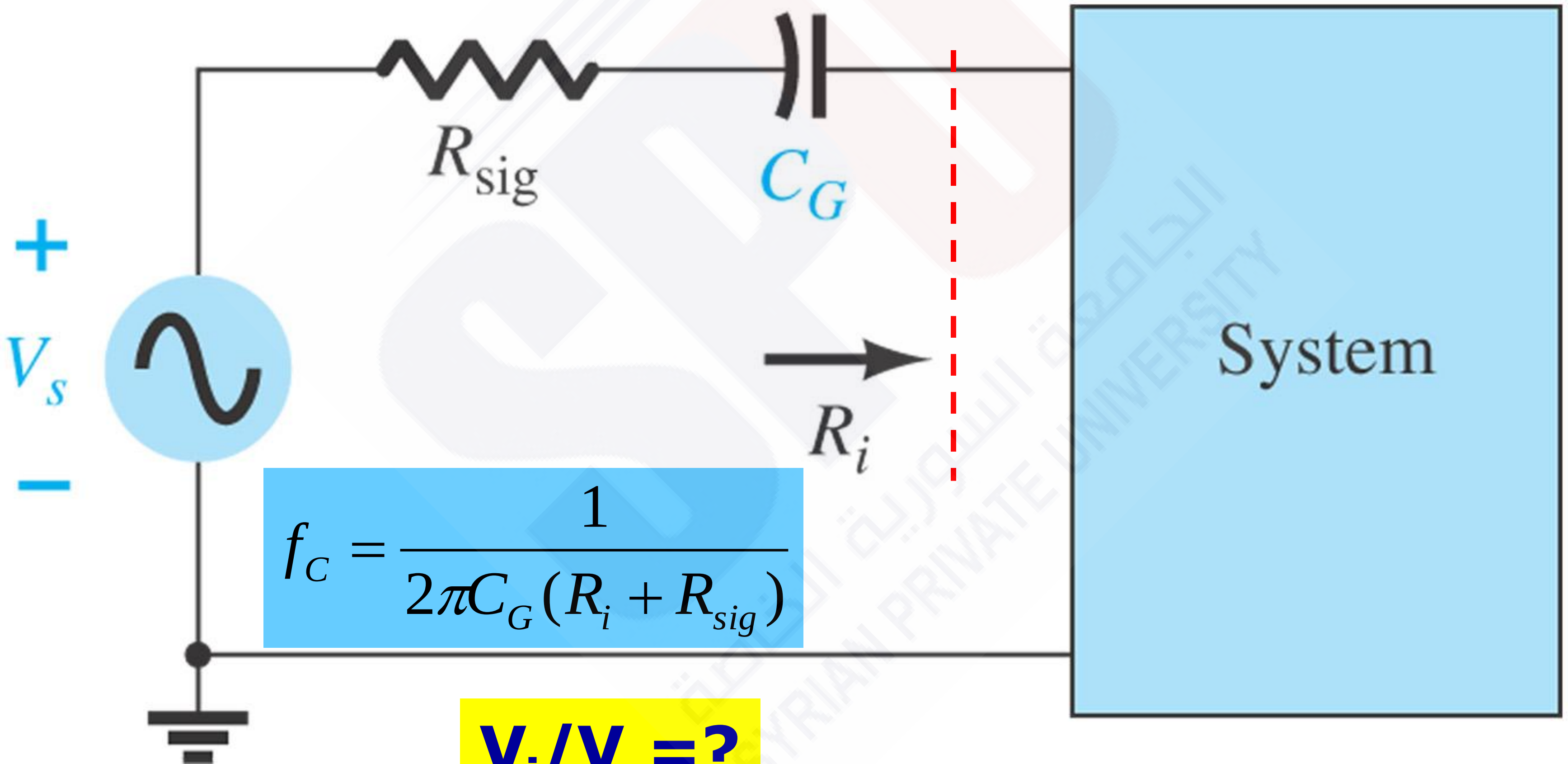
$V_E/V_s=?$

$$f_C = \frac{1}{2\pi C_E R_e}$$

3.4 Low Frequency Analysis-FET Amp.



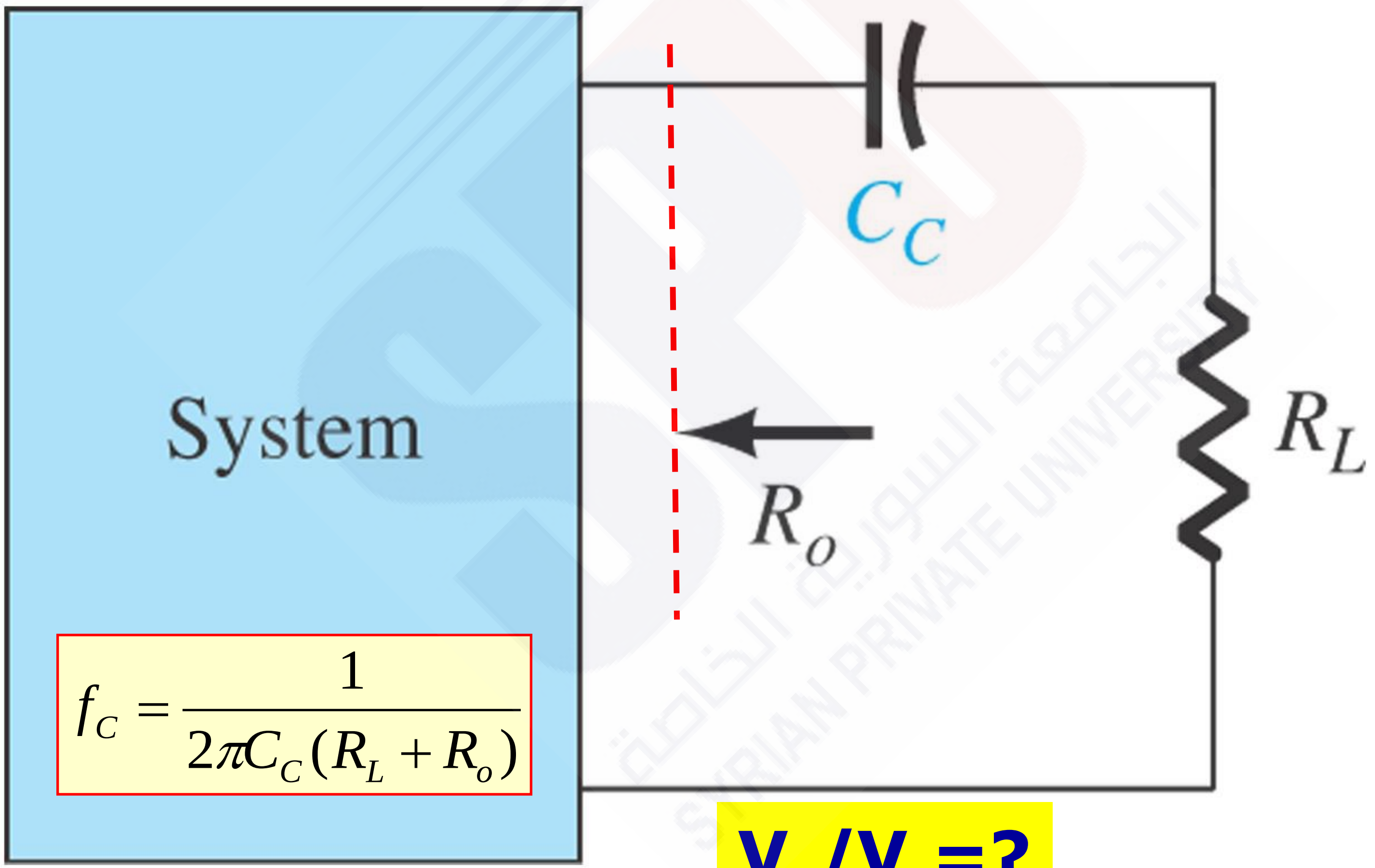
C_G Effect



$$f_C = \frac{1}{2\pi C_G (R_i + R_{sig})}$$

V_i/V_s=?

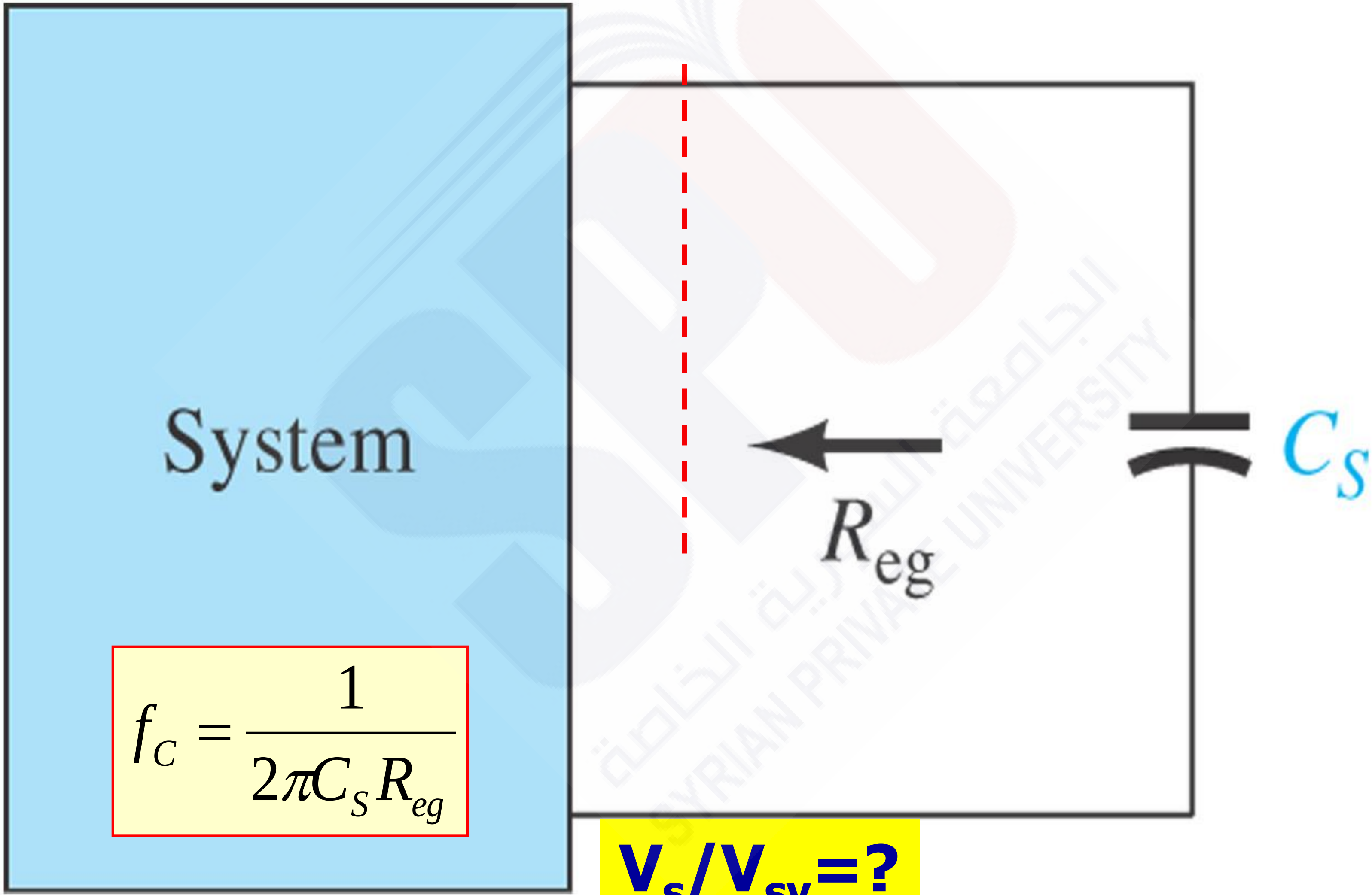
C_C Effect



$$f_C = \frac{1}{2\pi C_C (R_L + R_o)}$$

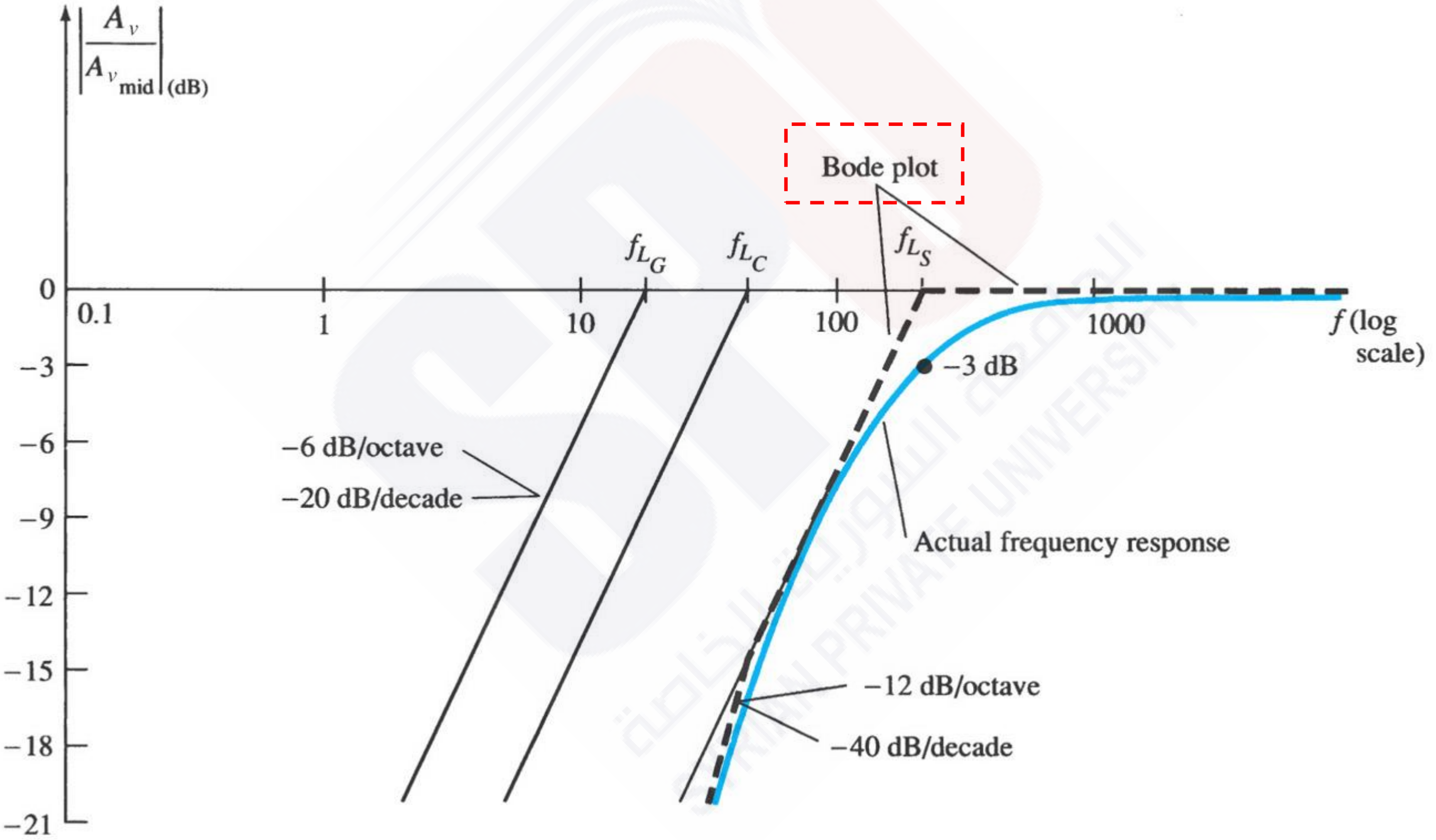
V_o/V_s=?

C_s Effect



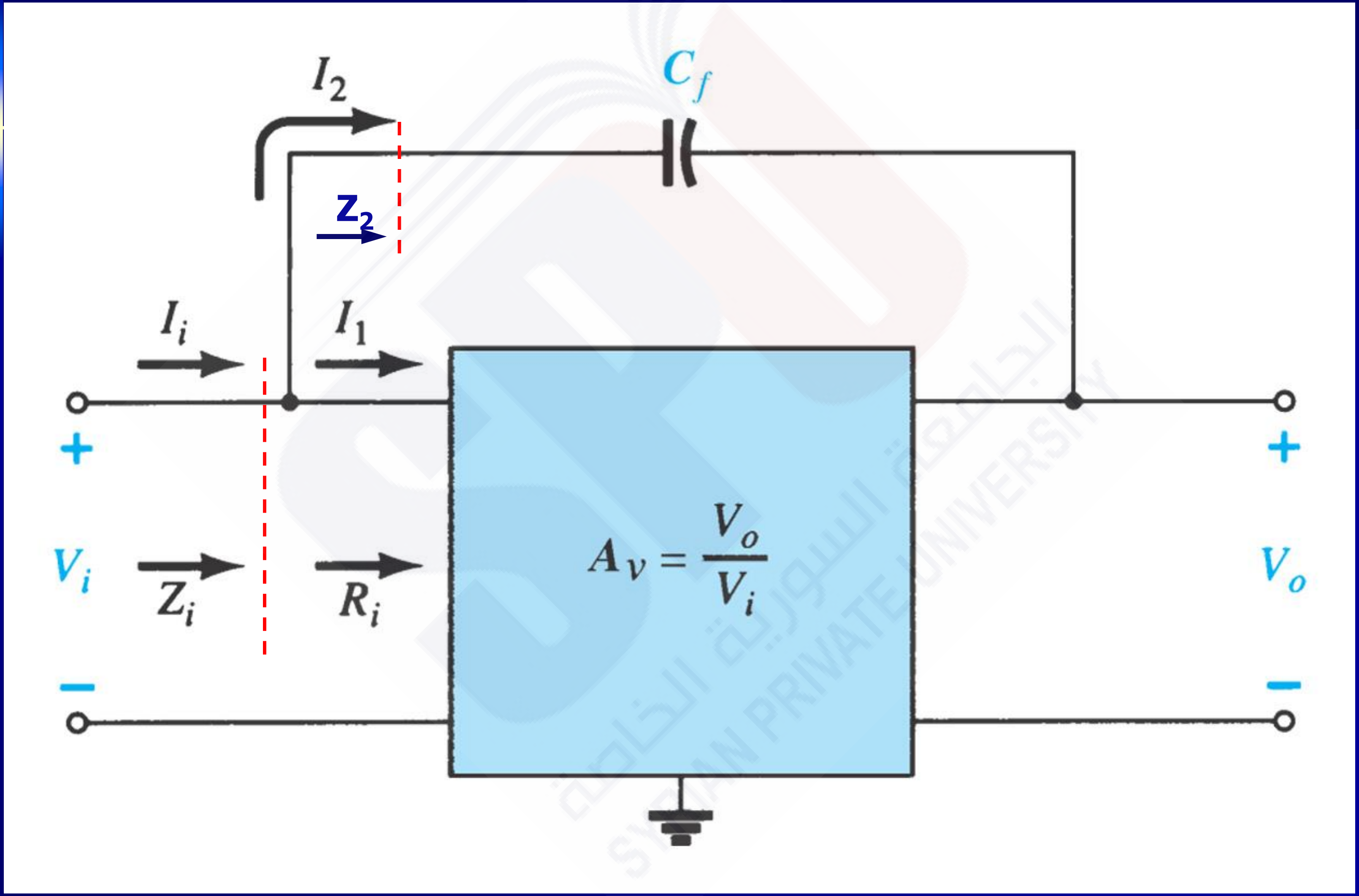
$$f_C = \frac{1}{2\pi C_S R_{eg}}$$

V_s/V_{sy}=?

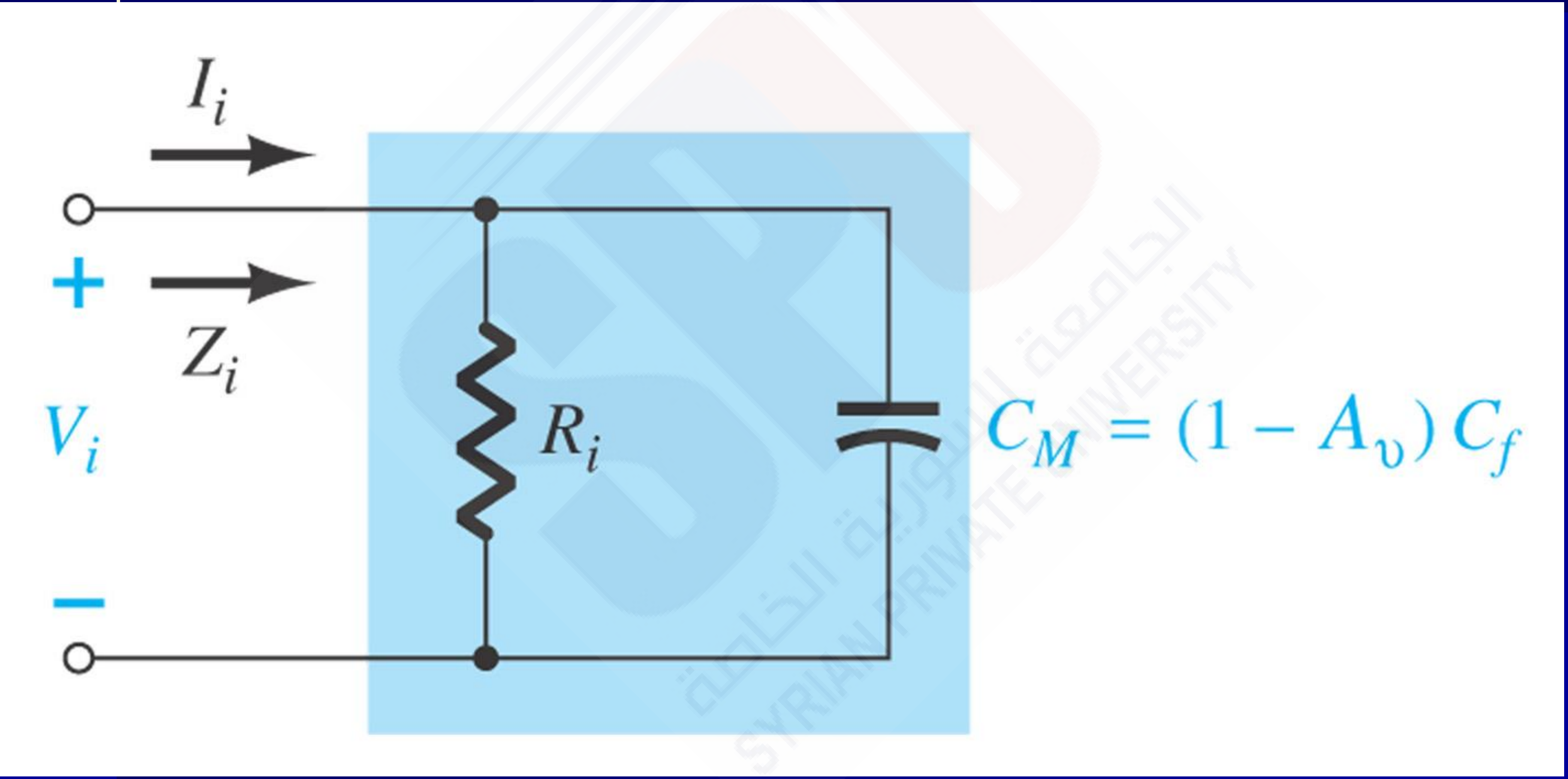


$\left|\frac{A_v}{A_{v_{mid}}}\right|_{(dB)}$
Bode plot
f_{L_G}
f_{L_C}
f_{L_S}
0
0.1
1
10
100
1000
f (log scale)
−3
−6
−9
−12
−15
−18
−21
−3 dB
−6 dB/octave
−20 dB/decade
Actual frequency response
−12 dB/octave
−40 dB/decade

Miller Effect



Miller Effect Calculation



Applying Kirchhoff's current law gives

$$I_i = I_1 + I_2$$

Using Ohm's law yields

$$I_i = \frac{V_i}{Z_i}, \quad I_1 = \frac{V_i}{R_i}$$

and
$$I_2 = \frac{V_i - V_o}{X_{C_f}} = \frac{V_i - A_v V_i}{X_{C_f}} = \frac{(1 - A_v)V_i}{X_{C_f}}$$

Substituting, we obtain

$$\frac{V_i}{Z_i} = \frac{V_i}{R_i} + \frac{(1 - A_v)V_i}{X_{C_f}}$$

and
$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{C_f}/(1 - A_v)}$$

but
$$\frac{X_{C_f}}{1 - A_v} = \frac{1}{\omega \underbrace{(1 - A_v)C_f}_{C_M}} = X_{C_M}$$

and
$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{C_M}}$$

The Miller effect will also increase the level of output capacitance, which must also be considered when the high-frequency cutoff is determined. In Fig. 9.45, the parameters of importance to determine the output Miller effect are in place. Applying Kirchhoff's current law results in

$$I_o = I_1 + I_2$$

with $$I_1 = \frac{V_o}{R_o} \quad \text{and} \quad I_2 = \frac{V_o - V_i}{X_{C_f}}$$

The resistance R_o is usually sufficiently large to permit ignoring the first term of the equation compared to the second term and assuming that

$$I_o \cong \frac{V_o - V_i}{X_{C_f}}$$

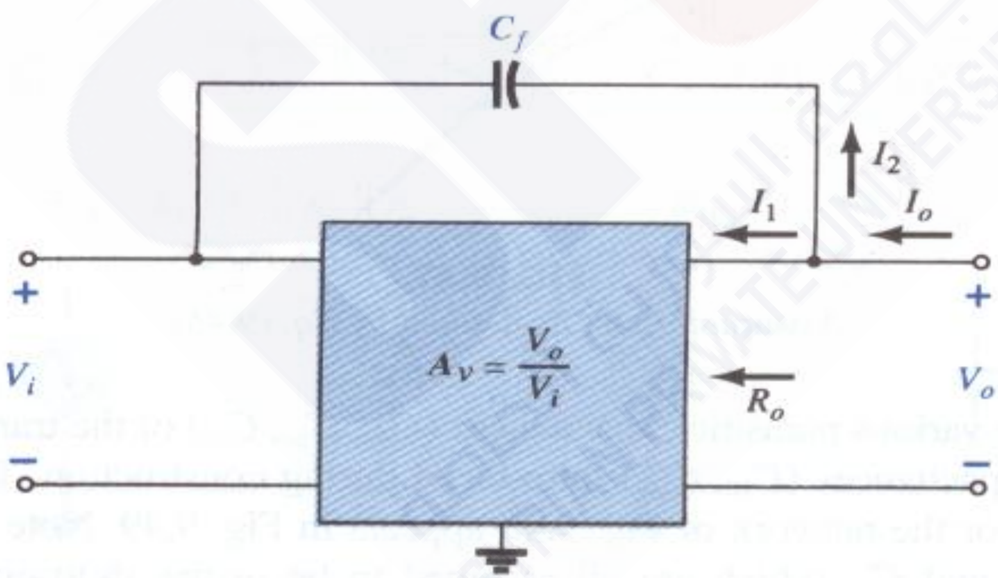


FIG. 9.45
Network employed in the derivation of an equation for the Miller output capacitance.

Substituting $V_i = V_o/A_v$ from $A_v = V_o/V_i$ results in

$$I_o = \frac{V_o - V_o/A_v}{X_{C_f}} = \frac{V_o(1 - 1/A_v)}{X_{C_f}}$$

and

$$\frac{I_o}{V_o} = \frac{1 - 1/A_v}{X_{C_f}}$$

or

$$\frac{V_o}{I_o} = \frac{X_{C_f}}{1 - 1/A_v} = \frac{1}{\omega C_f(1 - 1/A_v)} = \frac{1}{\omega C_{M_o}}$$

resulting in the following equation for the Miller output capacitance:

$$\boxed{C_{M_o} = \left(1 - \frac{1}{A_v}\right)C_f} \qquad \textbf{(9.43)}$$

For the usual situation where $A_v \gg 1$, Eq. (9.43) reduces to

$$\boxed{C_{M_o} \cong C_f}_{|A_v| >> 1} \qquad \textbf{(9.44)}$$

Examples of the use of Eq. (9.43) appear in the next two sections as we investigate the high-frequency responses of BJT and FET amplifiers.

9.9 HIGH-FREQUENCY RESPONSE—BJT AMPLIFIER

At the high-frequency end, there are two factors that define the −3-dB cutoff point: the network capacitance (parasitic and introduced) and the frequency dependence of $h_{fe}(\beta)$.

Network Parameters

In the high-frequency region, the *RC* network of concern has the configuration appearing in Fig. 9.46. At increasing frequencies, the reactance X_C will decrease in magnitude, resulting in a shorting effect across the output and a decrease in gain. The derivation leading to the corner frequency for this R_C configuration follows along similar lines to that encountered for the low-frequency region. The most significant difference is in the following general form of A_v:

$$\boxed{A_v = \frac{1}{1 + j(f/f_2)}} \qquad (9.45)$$

This results in a magnitude plot such as shown in Fig. 9.47 that drops off at 6 dB/octave with increasing frequency. Note that f_2 is in the denominator of the frequency ratio rather than the numerator as occurred for f_1 in Eq. (9.21).

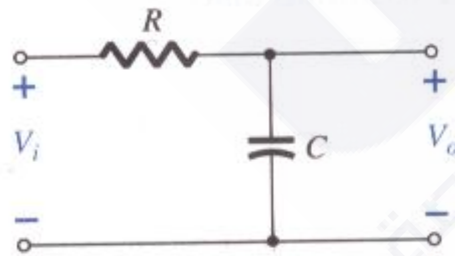


FIG. 9.46
RC combination that will define a high-cutoff frequency.

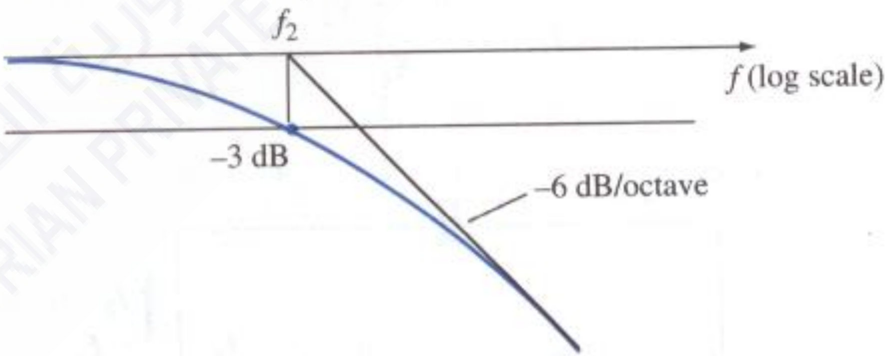


FIG. 9.47
Asymptotic plot as defined by Eq. (9.45).

$$\boxed{f_{H_i} = \frac{1}{2\pi R_{\mathrm{Th}_i} C_i}}$$

with

$$\boxed{R_{\mathrm{Th}_i} = R_s \| R_1 \| R_2 \| R_i}$$

and

$$\boxed{C_i = C_{W_i} + C_{be} + C_{M_i} = C_{W_i} + C_{be} + (1 - A_v)C_{bc}}$$

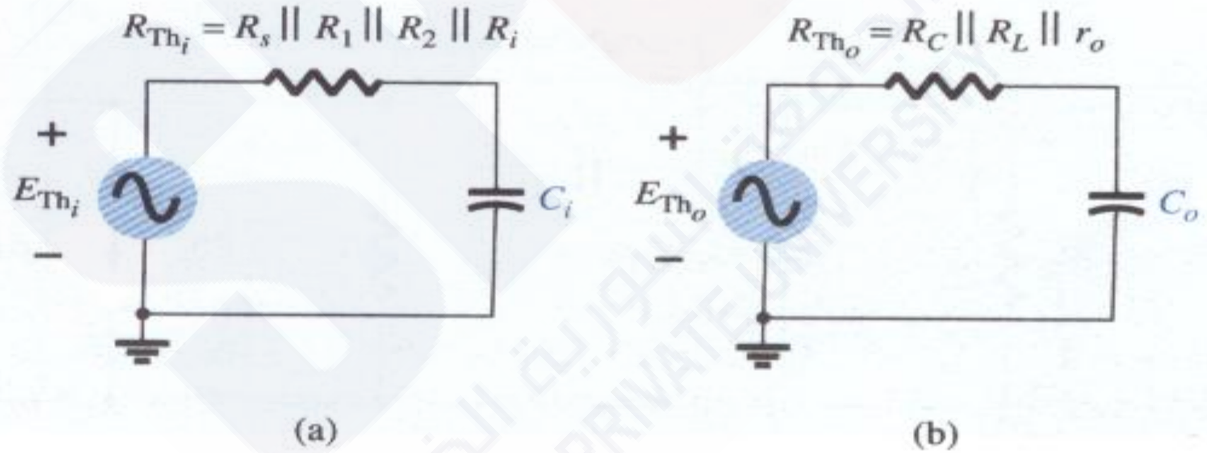
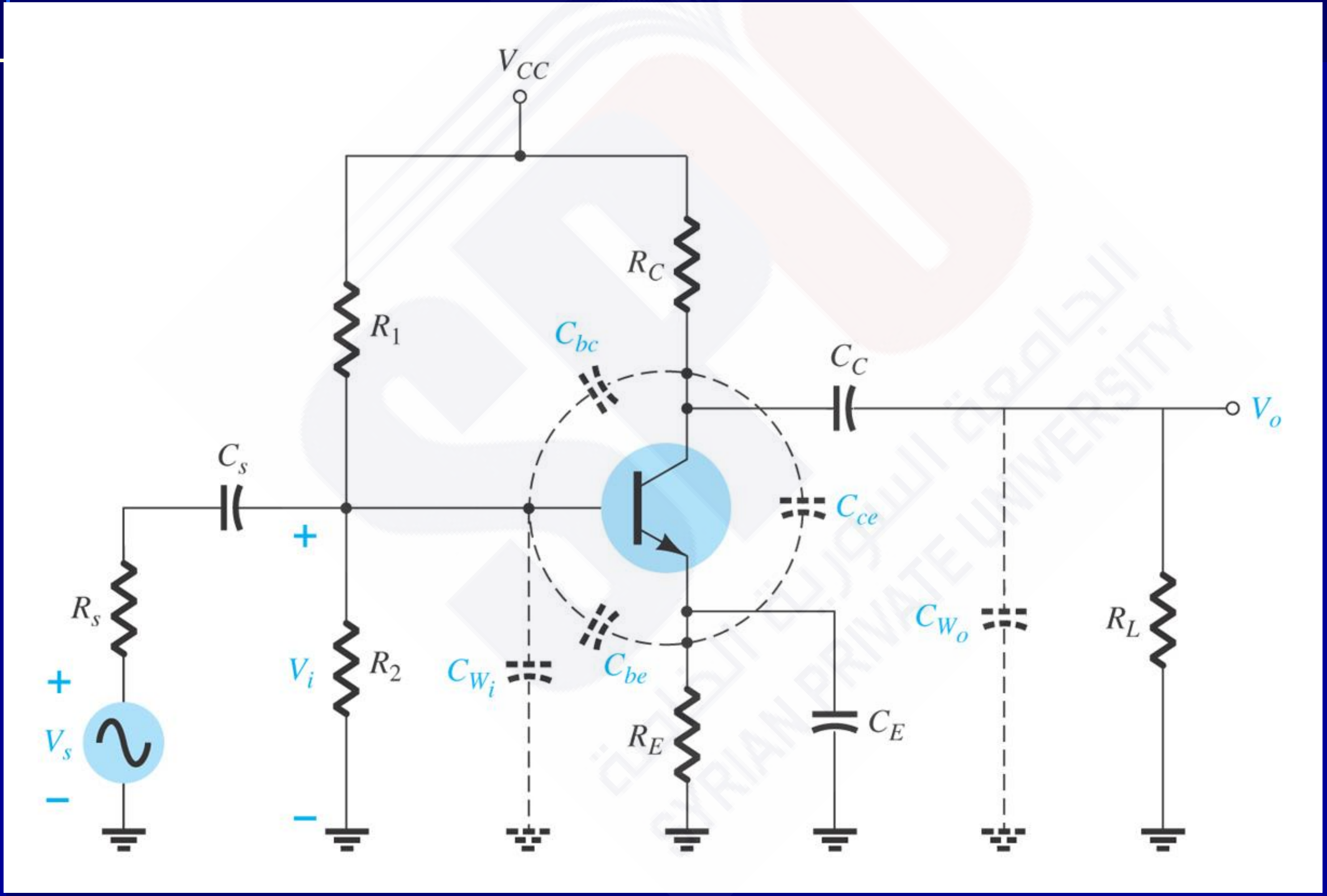


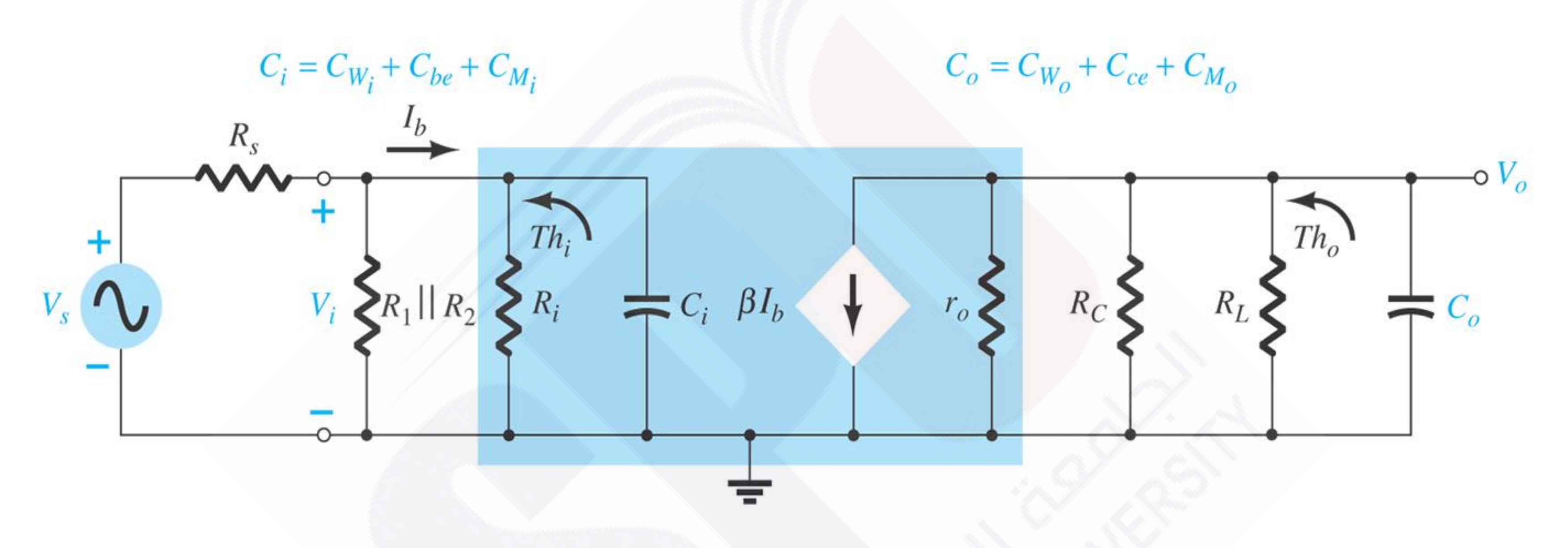
FIG. 9.50

Thévenin circuits for the input and output networks of the network of Fig. 9.49.

3.5 High Frequency Analysis-BJT Amp.



High Frequency Cut-off: f_C



$$f_C = \frac{1}{2\pi C_i (R_i \| R_1 \| R_2 \| R_s)}$$

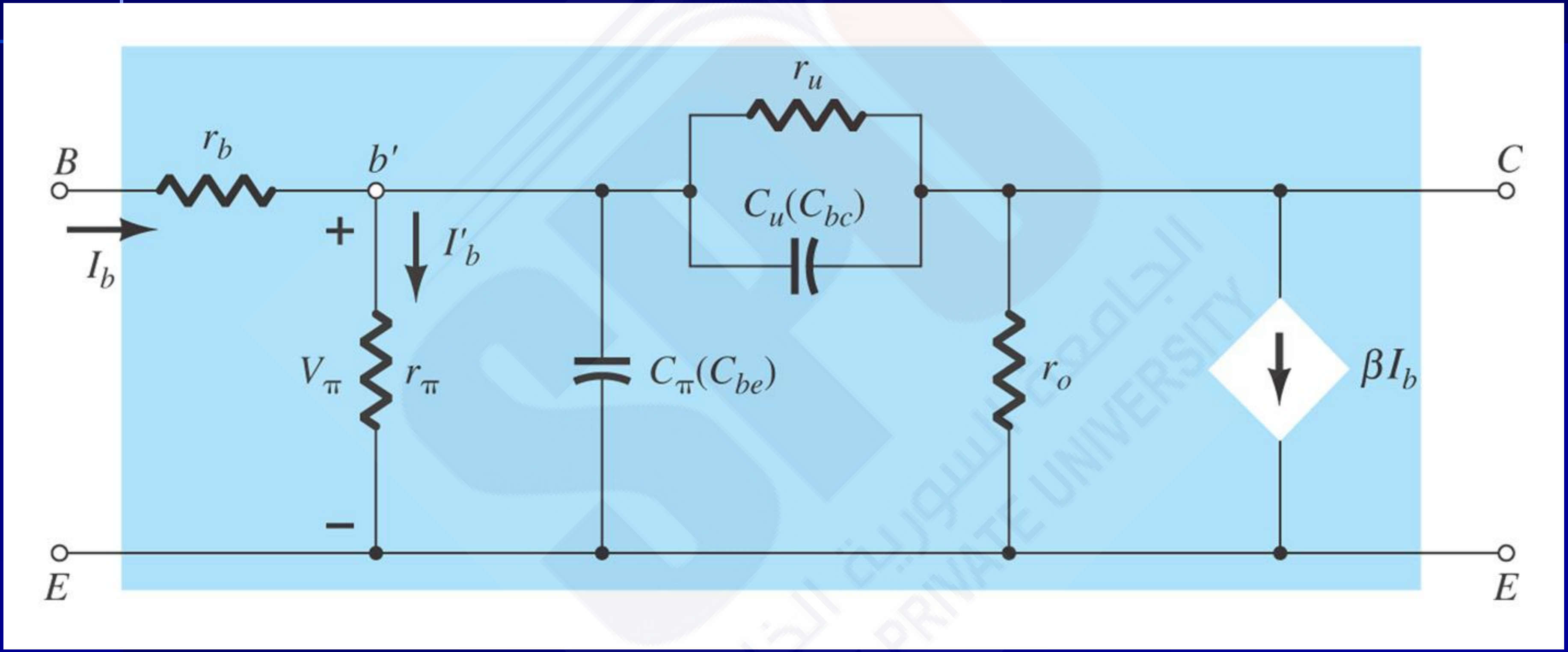
$$h_{fe} = \frac{h_{fe_{\text{mid}}}}{1 + j(f/f_\beta)}$$

$$f_{H_o} = \frac{1}{2\pi R_{\text{Th}_o} C_o}$$

$$R_{Th_o} = R_C \| R_L \| r_o$$

$$C_o = C_{W_o} + C_{ce} + C_{M_o}$$

Giageleto Equivalent Circuit



$$C_i = C_{be} + C_{bc}(1 + A_v)$$

$$\boxed{f_\beta \text{ (sometimes appearing as } f_{h_{fe}}) = \frac{1}{2\pi r_\pi (C_\pi + C_u)}}$$

or, since $r_\pi = \beta r_e = h_{fe_{mid}} r_e$,

$$\boxed{f_\beta = \frac{1}{h_{fe_{mid}}} \frac{1}{2\pi r_e (C_\pi + C_u)}}$$

or

$$\boxed{f_\beta \cong \frac{1}{2\pi \beta_{mid} r_e (C_\pi + C_u)}}$$

The following equation permits a direct conversion for determining f_β if f_α and α are specified:

$$\boxed{f_\beta = f_\alpha(1 - \alpha)} \tag{9.56}$$

A quantity called the *gain–bandwidth product* is defined for the transistor by the condition

$$\left|\frac{h_{fe_{\text{mid}}}}{1 + j(f/f_\beta)}\right| = 1$$

so that

$$|h_{fe}|_{\text{dB}} = 20\log_{10}\left|\frac{h_{fe_{\text{mid}}}}{1 + j(f/f_\beta)}\right| = 20\log_{10} 1 = 0\text{ dB}$$

The frequency at which $|h_{fe}|_{\text{dB}} = 0\text{ dB}$ is clearly indicated by f_T in Fig. 9.52. The magnitude of h_{fe} at the defined condition point ($f_T \gg f_\beta$) is given by

$$\frac{h_{fe_{\text{mid}}}}{\sqrt{1 + (f_T/f_\beta)^2}} \cong \frac{h_{fe_{\text{mid}}}}{f_T/f_\beta} = 1$$

so that

$$\boxed{f_T \cong \underbrace{h_{fe_{\text{mid}}} f_\beta}^{(\cong \text{BW})}} \quad \text{(gain–bandwidth product)} \tag{9.57}$$

or

$$\boxed{f_T \cong \beta_{\text{mid}}\, f_\beta} \tag{9.58}$$

with

$$\boxed{f_\beta = \frac{f_T}{\beta_{\text{mid}}}} \tag{9.59}$$

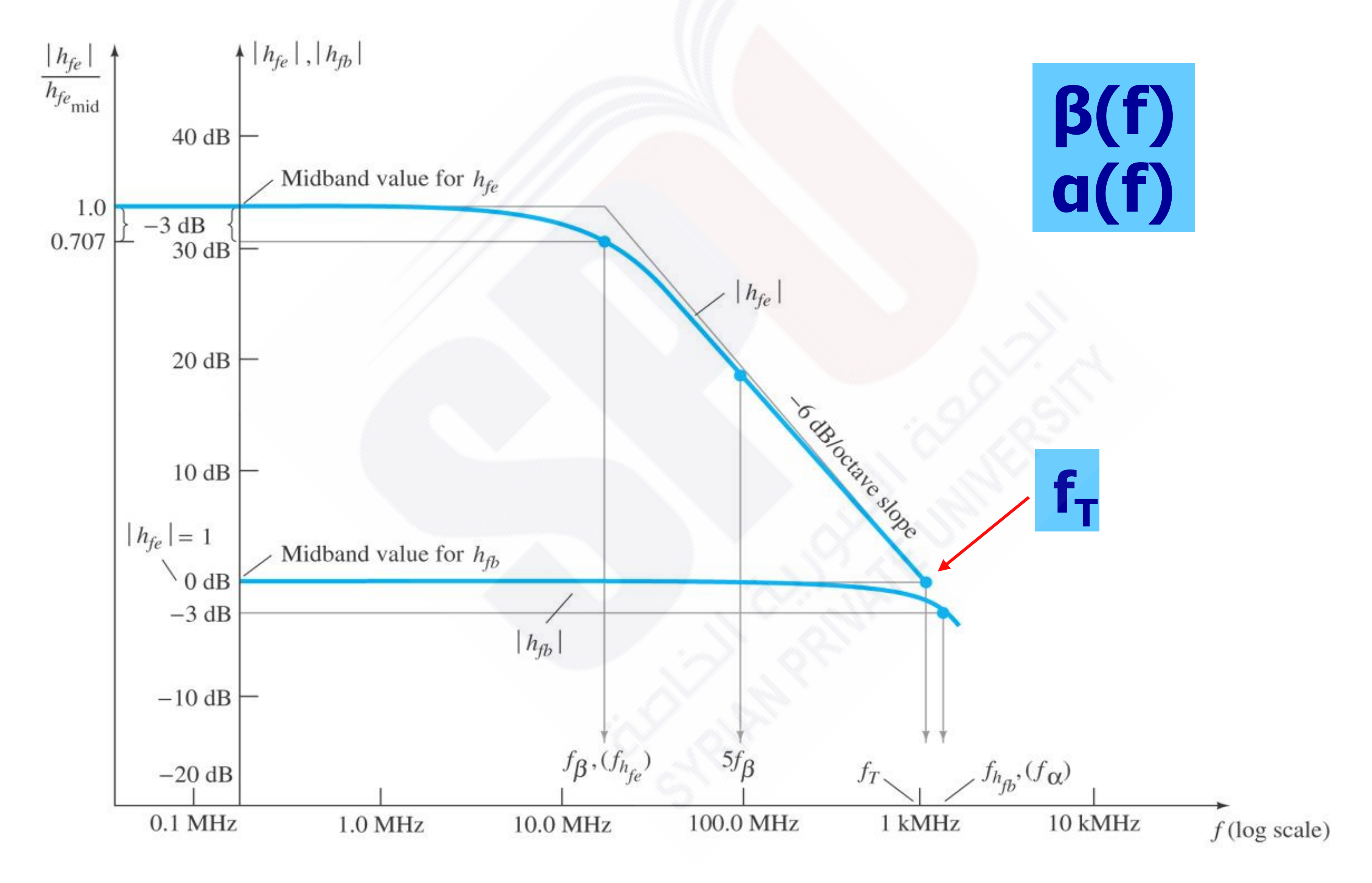
Substituting Eq. (9.55) for f_β in Eq. (9.57) gives

$$f_T \cong \beta_{\text{mid}} \frac{1}{2\pi\beta_{\text{mid}} r_e(C_\pi + C_u)}$$

and

$$\boxed{f_T \cong \frac{1}{2\pi r_e(C_\pi + C_u)}} \tag{9.60}$$

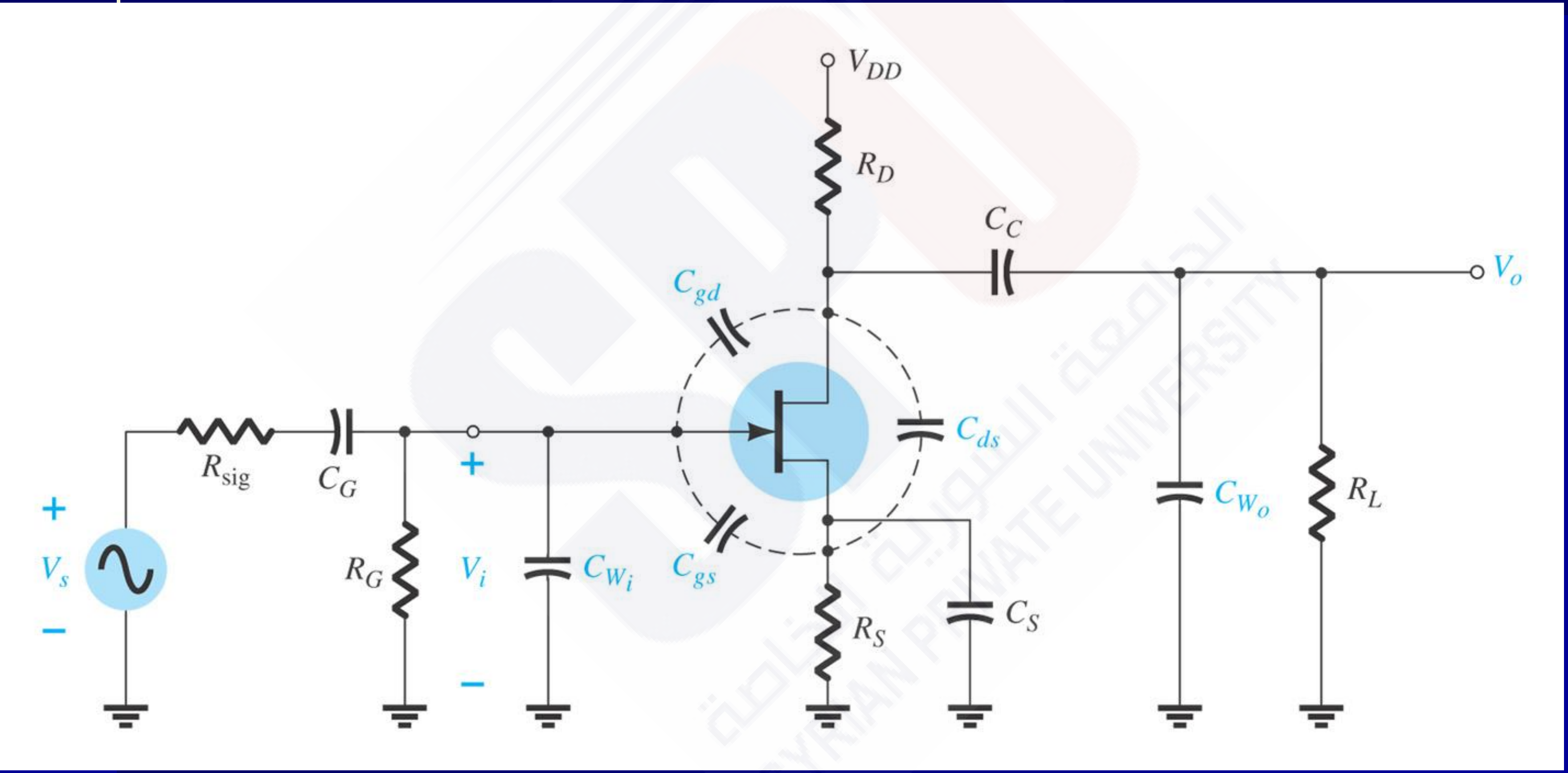
Bode-Plot at High Frequency



β(f)
α(f)

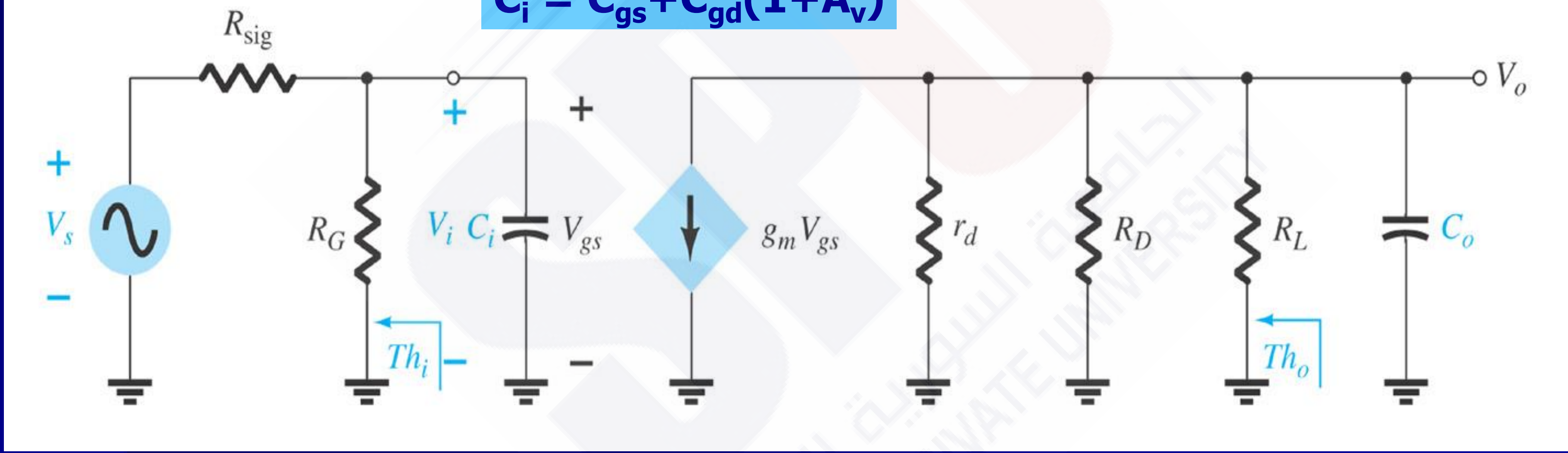
f_T

3.6 High Frequency Analysis-FET Amp.



High Frequency Equivalent Circuit of FET

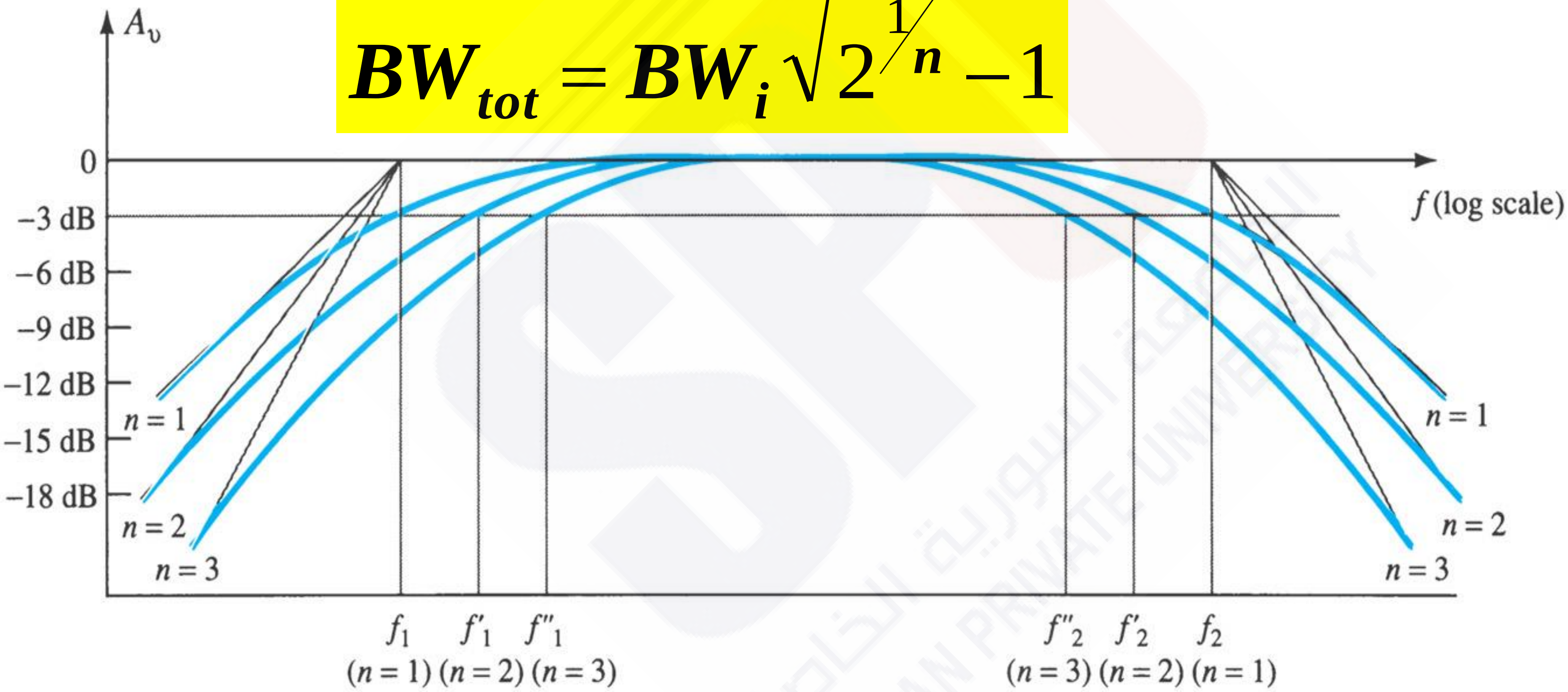
$C_i = C_{gs} + C_{gd}(1 + A_v)$

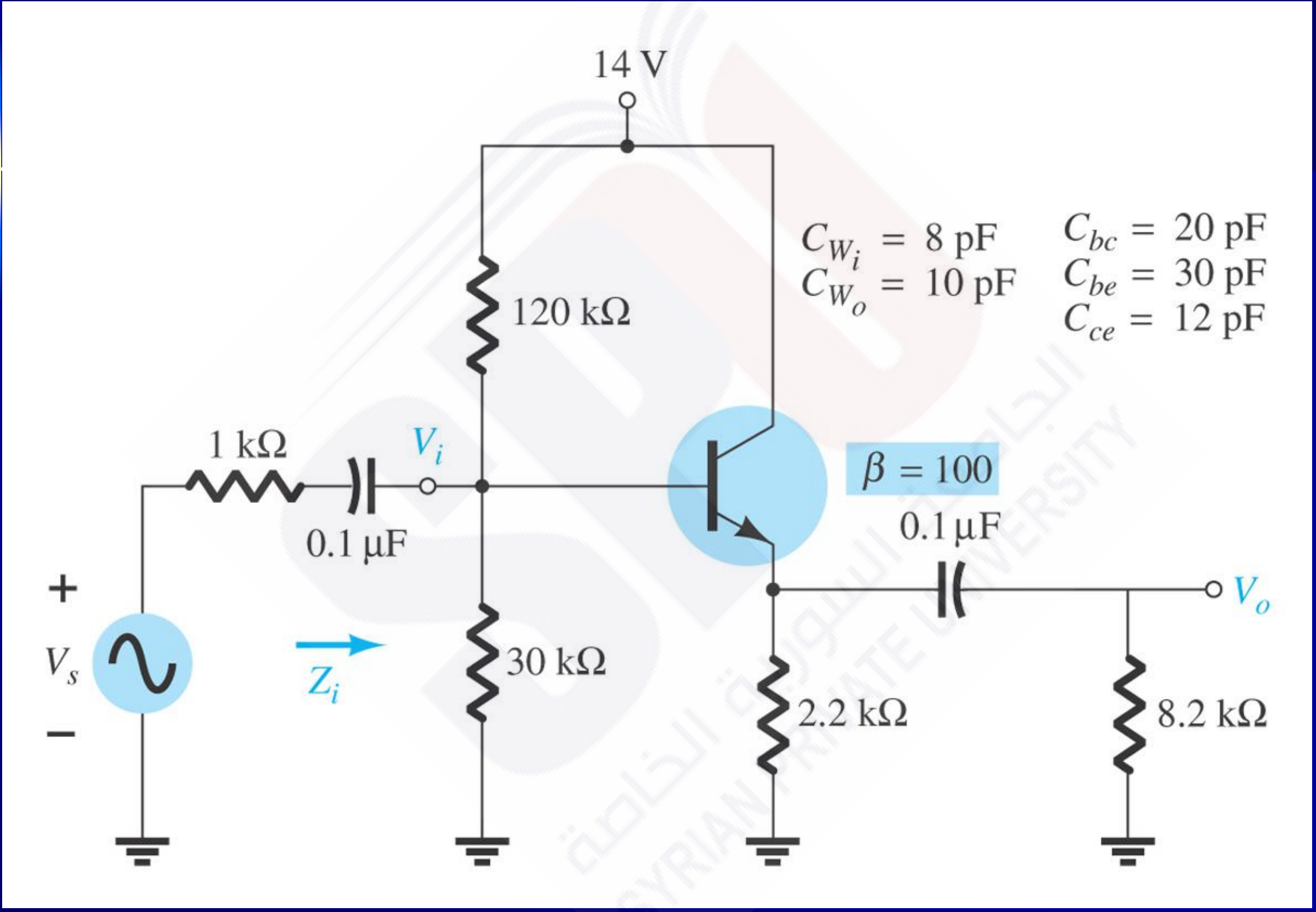


$$f_C = \frac{1}{2\pi C_i (R_G \| R_{sig})}$$

3.7 Multi-stages Frequency Effects

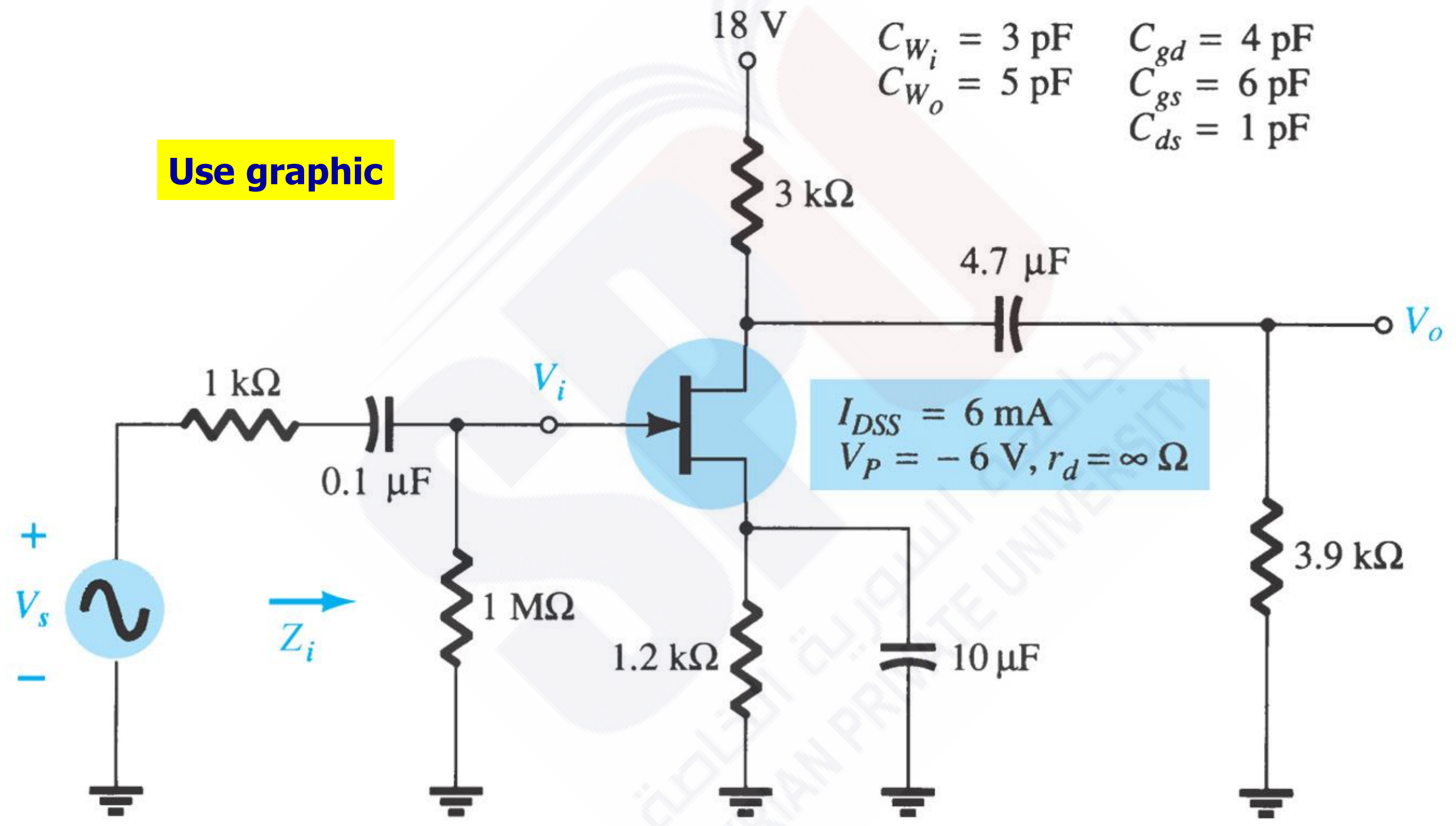
$$BW_{tot} = BW_i \sqrt{2^{1/n} - 1}$$





14 V
C_{W_i} = 8 pF
C_{W_o} = 10 pF
C_{bc} = 20 pF
C_{be} = 30 pF
C_{ce} = 12 pF
120 kΩ
1 kΩ
V_i
0.1 μF
β = 100
0.1 μF
+
V_s
−
Z_i
30 kΩ
V_o
2.2 kΩ
8.2 kΩ

Use graphic



18 V
$C_{W_i} = 3\text{ pF}$
$C_{W_o} = 5\text{ pF}$
$C_{gd} = 4\text{ pF}$
$C_{gs} = 6\text{ pF}$
$C_{ds} = 1\text{ pF}$
3 kΩ
4.7 μF
V_o
1 kΩ
V_i
0.1 μF
$I_{DSS} = 6\text{ mA}$
$V_P = -6\text{ V}, r_d = \infty\ \Omega$
+
V_s
−
Z_i
1 MΩ
1.2 kΩ
10 μF
3.9 kΩ